Presented at Short Course III on Exploration for Geothermal Resources,
organized by UNU-GTP and KenGen, at Lake Naivasha, Kenya, October 24- November 17, 2008.

UNITED NATIONS UNIVERSITY
GEOTHERMAL TRAINING PROGRAMME

KenGen
Kenya Electricity Generating Co., Ltd.

THE MAGNETO TELLURIC (MT) METHOD

Knútur Árnason
ISOR - Iceland GeoSurvey
Grensásvegur 9
108 Reykjavík
ICELAND
ka@isor.is

ABSTRACT

In this article the main aspects of the Magneto Telluric (MT) method is introduced. How the resistivity is calculated is derived from basic electromagnetic theory and then it is applied to practical situations, horizontally layered earth, solved for 1D and 2D structure.

1. INTRODUCTION

In the MT-method time variations in the Earth's magnetic field are used to probe the subsurface resistivity structure.

The Earth's electromagnetic field contains a wide frequency spectrum (Figure 1). The low frequencies are generated by ionospheric and magnetospheric currents caused by solar wind (plasma) interfering

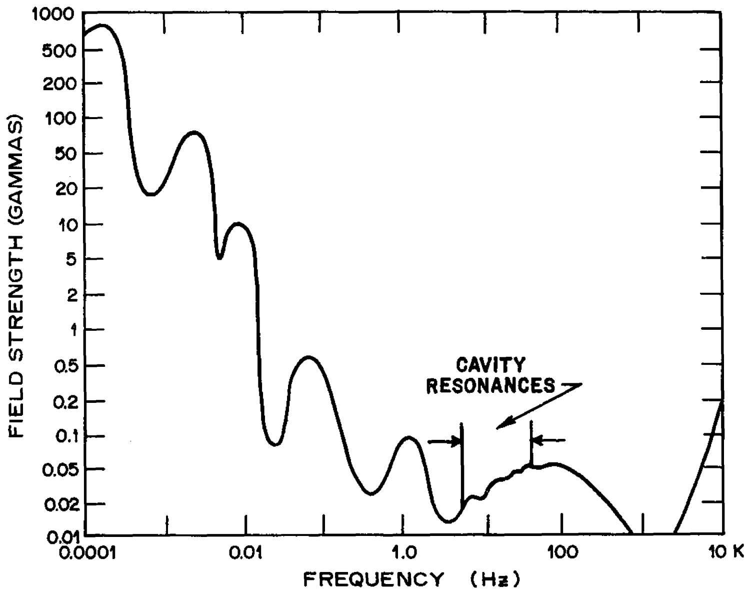

Figure 1: Natural magnetic field spectrum.

with the Earth's magnetic field. Higher frequencies ($> 1\ Hz$) are due to thunderstorms near equator distributed as guided waves between the Earth and the ionosphere. The time varying magnetic field induces electric field and hence currents in the ground. By measuring variations in the magnetic and electric fields in the surface of the ground, information about the subsurface resistivity structure can be obtained.

As we will see, the depth of penetration of the electromagnetic field depends on frequency. Low frequency variations penetrate, and hence probe, deep into the Earth but high frequency variations probe shallow depths.

In the MT terminology a distinction is made between applied frequency ranges. When variations of frequencies lower than $10\ Hz$ are used we speak of MT-method but for frequencies higher than $10\ Hz$ we speak of AMT (Audiofrequency Magneto Telluric) -method. This distinction is partly based on different source mechanism, but mainly because of difference in measuring instruments.

1.1. Basic Electromagnetic Theory

Maxwell's equations in conducting medium

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon} \eta \tag{4}$$

where

$$\mathbf{B} = \mu \mathbf{H}; \qquad \mu = \mu_r \mu_0; \qquad \varepsilon = \varepsilon_r \varepsilon_0$$
$$\mu_0 = 4\pi \cdot 10^{-7}\ {}^{Henry}\!/_{m}; \qquad \varepsilon_0 = 8.85 \cdot 10^{-12}\ {}^{Farad}\!/_{m}$$

In the Earth we have usually that

$$\mu_r \simeq 1 \text{ and } \varepsilon_r \simeq 1$$

Outside discontinuities in the conductivity σ we have no charge density η so that $\nabla \cdot \mathbf{E} = 0$.

By taking time derivative of (2) and by multiplying by μ and using (1) we get

$$-\nabla \times (\nabla \times \mathbf{E}) = \mu\sigma \frac{\partial}{\partial t}\mathbf{E} + \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{E}$$

using that

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla \cdot (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

and that $\nabla \cdot \mathbf{E} = 0$ we get

$$\nabla^2 \mathbf{E} - \mu\sigma \frac{\partial}{\partial t}\mathbf{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{E} = 0 \tag{5}$$

By similar procedure we get

$$\nabla^2 \mathbf{H} - \mu\sigma \frac{\partial}{\partial t}\mathbf{H} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{H} = 0 \tag{6}$$

For $\sigma = 0$, Equations (5) and (6) are wave equations describing waves propagating with velocity $v = \frac{1}{\sqrt{\varepsilon\mu}}$. For high conductivities (low resistivities, $\rho = \frac{1}{\sigma}$) Equations (5) and (6) are diffusion equations.

If the fields vary harmonically in time, that is as $e^{i\omega t}$ we get:

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0; \qquad \nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0 \tag{7}$$

where

$$k^2 = \mu\varepsilon\omega^2 - i\mu\sigma\omega \tag{8}$$

Equations (7) have plain wave solutions

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}^{+} e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + \mathbf{E}^{-} e^{i(\mathbf{k}\cdot\mathbf{x}+\omega t)} \tag{9}$$

$$\mathbf{H}(\mathbf{x}, t) = \mathbf{H}^{+} e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + \mathbf{H}^{-} e^{i(\mathbf{k}\cdot\mathbf{x}+\omega t)} \tag{10}$$

where $\mathbf{k} = k\mathbf{u}$ is the propagation vector and $\mathbf{u}$ is normal to the planes of constant phase. The first terms in (9) and (10) is a wave propagating in the direction of $\mathbf{u}$ but the second terms, a wave propagating in the opposite direction.

The wave number k is the square root of (8) and is complex;

$$k = \alpha - i\beta \tag{11}$$

where

$$\alpha = \sqrt{\varepsilon\mu}\,\omega\sqrt{\frac{1}{2}\left(\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right)} \tag{12}$$

$$\beta = \sqrt{\varepsilon\mu}\,\omega\sqrt{\frac{1}{2}\left(\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right)} \tag{13}$$

For the electric field of a wave propagating in the direction of $\mathbf{u}$ we have

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}^{+} e^{-\beta\mathbf{u}\cdot\mathbf{x}} e^{-i(\alpha\mathbf{u}\cdot\mathbf{x}-\omega t)} \tag{14}$$

which is a wave propagating with velocity

$$v = \frac{\omega}{\alpha} \tag{15}$$

and exponentially decreasing in amplitude. For $\sigma = 0$ we see from (12) and (13) that $\beta = 0$ and hence no decrease in amplitude and $\alpha = \sqrt{\varepsilon\mu}\,\omega$ so that

$$v_0 = \frac{1}{\sqrt{\varepsilon\mu}} \tag{16}$$

The resistivity of ground rocks is usually in the range of $1-10^4\ \Omega m$ so that σ is in the range $1-10^{-4}\ {}^{S}/_{m}$. The frequencies used in MT are lower than $10^3\ Hz$ so we have, since $\varepsilon_r \simeq 1$,

$$\left(\frac{\sigma}{\varepsilon\omega}\right)^2 > \left(\frac{10^{-4}}{8.85\cdot 10^{-12}\cdot 2\pi\cdot 10^3}\right)^2 = 3.2\cdot 10^6 >> 1 \tag{17}$$

We see therefore that we can take

$$\alpha \simeq \beta \simeq \sqrt{\frac{\mu\omega\sigma}{2}} = \frac{1}{\delta} \tag{18}$$

which is called the quasistationary approximation. δ is called the skindepth. From (14) we see that the amplitude of wave along the z-axis decreases as

$$E(z) \propto e^{-z/\delta} \tag{19}$$

and it is seen that the amplitude decreases faster for higher conductivities and frequencies. From (15) and (18) we see that the velocity is given

$$v = \sqrt{\frac{2\omega}{\mu\sigma}} \tag{20}$$

and decreases with increasing conductivity.

For a wave propagating in the direction of $\mathbf{u}$ with the wave vector $\mathbf{k} = k\mathbf{u}$, we have from (9) and (10)

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)}; \qquad \mathbf{H}(\mathbf{x}, t) = \mathbf{H}_0 e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{21}$$

From Maxwell's equations (3) and (4)

$$\left(\boldsymbol{\nabla}\cdot\mathbf{B} = \mu\boldsymbol{\nabla}\cdot\mathbf{H} = 0 \quad \text{and} \quad \boldsymbol{\nabla}\cdot\mathbf{E} = \frac{1}{\varepsilon}\eta = 0\right)$$

we see that

$$\mathbf{k}\cdot\mathbf{E} = k\mathbf{u}\cdot\mathbf{E} = 0; \qquad \mathbf{k}\cdot\mathbf{H} = k\mathbf{u}\cdot\mathbf{H} = 0$$

so both $\mathbf{E}$ and $\mathbf{H}$ are perpendicular to the wave vector $\mathbf{k}$.

From Maxwell's equation (2)

$$\left(\boldsymbol{\nabla}\times\mathbf{H} = \sigma\mathbf{E} + \varepsilon\frac{\partial}{\partial t}\mathbf{E}\right)$$

we see that

$$-i\mathbf{k}\times\mathbf{H} = (\sigma + i\varepsilon\omega)\mathbf{E} = \frac{ik^2}{\mu\omega}\mathbf{E} \tag{22}$$

or

$$\mathbf{E} = -\frac{\mu\omega}{k}\mathbf{u}\times\mathbf{H} \tag{23}$$

If we take z-axis along $\mathbf{u}$ then

$$\mathbf{u} = (0, 0, 1); \qquad \mathbf{u}\times\mathbf{H} = (-H_y, H_x, 0) \tag{24}$$

and we have

$$E_x = \frac{\mu\omega}{k}H_y; \qquad E_y = -\frac{\mu\omega}{k}H_x \tag{25}$$

which can be written

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \tag{26}$$

where

$$Z_{xy} = -Z_{yx} = Z = \frac{\mu\omega}{k} \tag{27}$$

In the quasistationary approximation $k = \sqrt{\mu\omega\sigma}\,\frac{1}{\sqrt{2}}\,(1-i)$ and we have

$$Z = \sqrt{\frac{\mu\omega}{\sigma}}\frac{\sqrt{2}}{1-i} = \sqrt{\frac{\mu\omega}{\sigma}}\frac{1+i}{\sqrt{2}} = \sqrt{\frac{\mu\omega}{\sigma}}e^{i\pi/4} \tag{28}$$

By measuring $\mathbf{E}$ and $\mathbf{H}$ we can determine the resistivity, because from (26) and (28) we can see that

$$\rho = \frac{1}{\sigma} = \frac{1}{\mu\omega}\left|\frac{E_x}{H_y}\right|^2 = \frac{1}{\mu\omega}\left|\frac{E_y}{H_x}\right|^2 \tag{29}$$

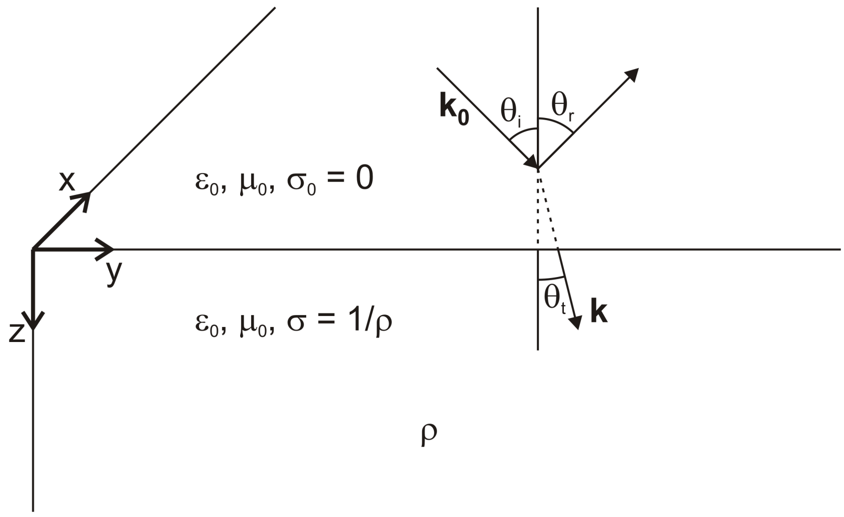

Figure 2: Reflected and refracted waves.

2. THE MT METHOD

We now turn to the MT-method and consider plain electromagnetic wave of angular frequency ω and wave vector $\mathbf{k_0}$ incident at the surface of a homogeneous earth with resistivity $\rho = 1/\sigma$. The wave vector $\mathbf{k_0}$ makes the angle θ_i (angle of incidence) with the z-axis. A refracted wave propagates into the half-space with wave vector $\mathbf{k}$ making the angle θ_t with the z-axis.

By Snell's law we have

$$\frac{1}{\upsilon_0}\sin\theta_i = \frac{1}{\upsilon}\sin\theta_t \tag{30}$$

where υ_0 and υ are the velocities in the air and the half-space respectively. From (16) and (20) we have

$$\upsilon_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}; \qquad \upsilon = \sqrt{\frac{2\omega}{\mu_o\sigma}} \tag{31}$$

and we have

$$\sin\theta_t = \sin\theta_i\sqrt{\frac{2\varepsilon_0\omega}{\sigma}} \tag{32}$$

In (17) we found that for $\rho = 1/\sigma < 10^4\ \Omega m$ and $\omega < 10^3\ Hz$

$$\frac{2\varepsilon_0\omega}{\sigma} < 10^{-7}$$

So that θ_t is practically zero and the refracted wave in the Earth has the wave vector $\mathbf{k}$ along the z-axis for all angles of incidence. The equations that we have derived (Equations (25)-(29)) apply to the transmitted wave. By measuring the $\mathbf{E}$ and $\mathbf{H}$ fields in the surface of the half-space we obtain the resistivity by Equation (29).

3. HORIZONTALLY LAYERED EARTH

In the surface of a horizontally layered earth we have

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \begin{pmatrix} 0 & Z_{xy}(\omega) \\ Z_{yx}(\omega) & 0 \end{pmatrix} \begin{pmatrix} H_x(\omega) \\ H_y(\omega) \end{pmatrix} \tag{33}$$

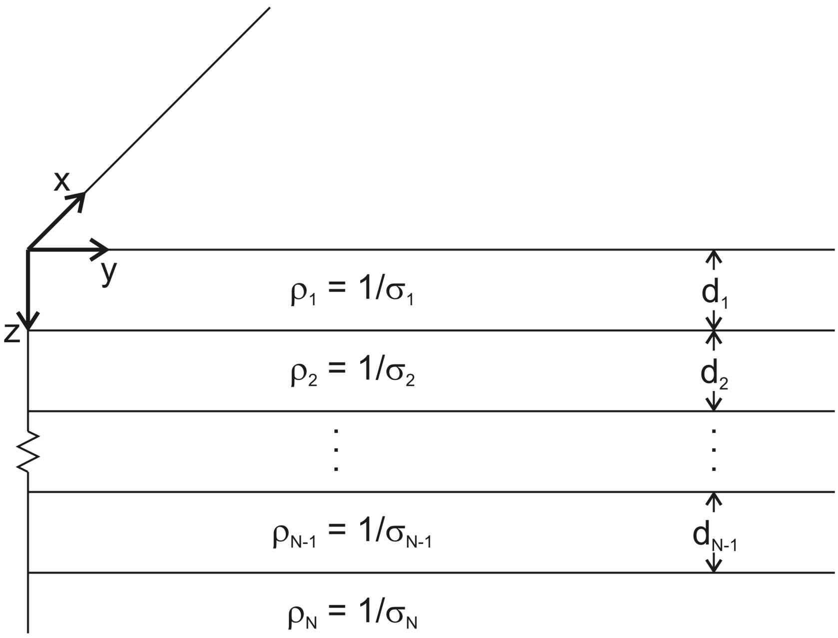

Figure 3: Layered earth.

where the impedance tensor elements are

$$Z_{xy}(\omega) = -Z_{yx}(\omega) = \hat{Z}_1 \tag{34}$$

The quantity $\hat{Z}_1$ is determined by a recursion relation

$$\hat{Z}_{\mathrm{i}} = Z_{\mathrm{i}} \frac{\hat{Z}_{\mathrm{i}+1} + Z_{\mathrm{i}} \tanh(ik_{\mathrm{i}}d_{\mathrm{i}})}{Z_{\mathrm{i}} + \hat{Z}_{\mathrm{i}+1} \tanh(ik_{\mathrm{i}}d_{\mathrm{i}})} \tag{35}$$

for i $= N-1,\ \dots,\ 1$ and $\hat{Z}_N = Z_N$

$$Z_{\mathrm{i}} = \frac{\mu_0 \omega}{k_{\mathrm{i}}}; \qquad k_{\mathrm{i}} = \sqrt{\mu_0 \varepsilon_0 \omega^2 - i\mu_0 \sigma_{\mathrm{i}} \omega} \tag{36}$$

The tensor element $\hat{Z}_1$ can be written

$$\hat{Z}_{\mathrm{i}}(\omega) = |\hat{Z}_{\mathrm{i}}(\omega)| e^{i\phi(\omega)} \tag{37}$$

where both $|\hat{Z}_{\mathrm{i}}|$ and ϕ are dependent on the resistivity layering and angular frequency.

In analogy with homogeneous half-space we define apparent resistivity

$$\rho_a(\omega) = \frac{1}{\mu_0 \omega} |\hat{Z}_1|^2 = \frac{1}{\mu_0 \omega} \left| \frac{E_x}{H_y} \right|^2 = \frac{1}{\mu_0 \omega} \left| \frac{E_y}{H_x} \right|^2 \tag{38}$$

which is a function of ω.

From the concept of skindepth, it is plausible that with lower frequencies we probe deeper into the Earth.

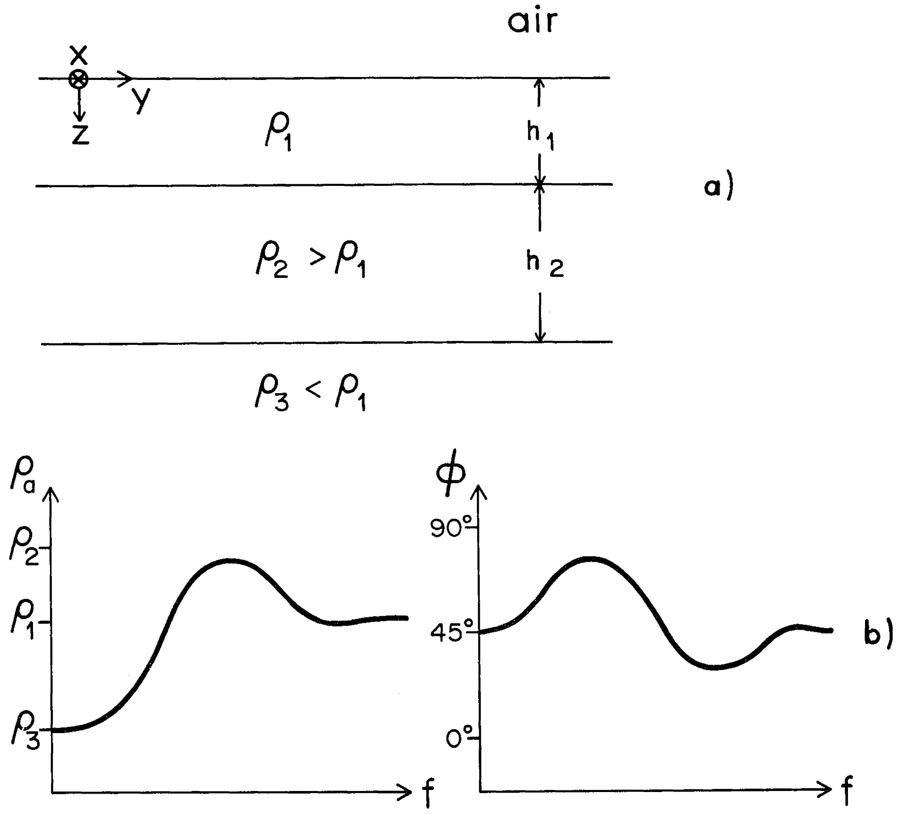

Figure 4: 1D MT apparent resistivity and phase.

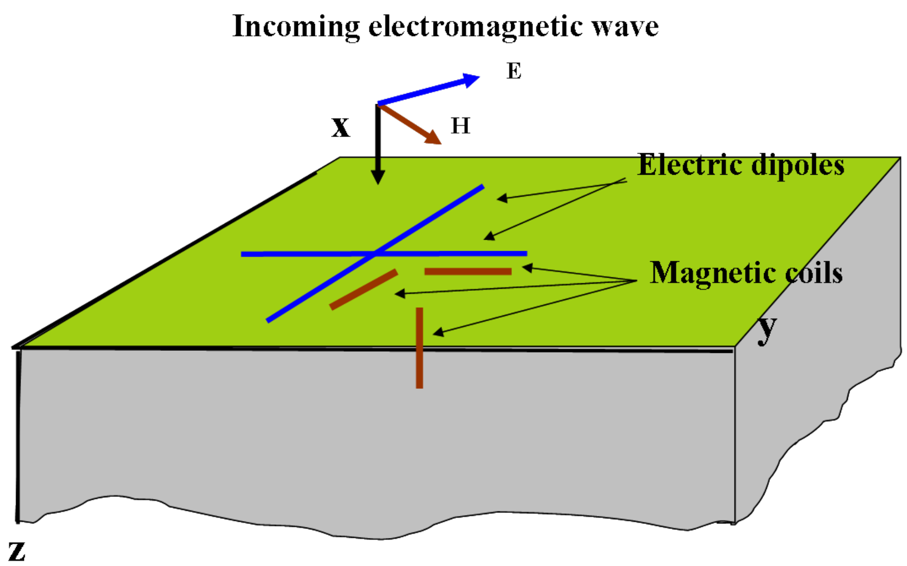

Figure 5: MT setup.

4. FIELD MEASUREMENT

We measure the electric and magnetic fields as a function of time in two perpendicular directions x and y

$$
\begin{gathered}
H_x(t); \qquad H_y(t) \\
E_x(t) = \frac{V_x(t)}{\Delta x}; \qquad E_y(t) = \frac{V_y(t)}{\Delta y}
\end{gathered} \tag{39}
$$

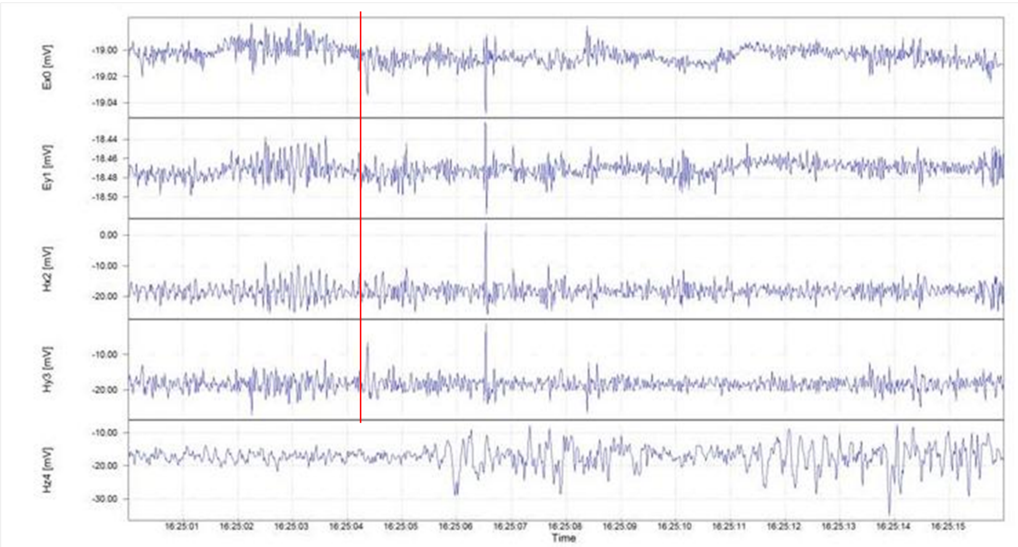

Figure 6: Timeseries.

Generally we have

$$
\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x(\omega) \\ H_y(\omega) \end{pmatrix} \tag{40}
$$

To determine the tensor elements we Fourier transform the timeseries

$$
\begin{gathered}
H_x(t) \rightarrow H_x(\omega); \qquad H_y(t) \rightarrow H_y(\omega) \\
E_x(t) \rightarrow E_x(\omega); \qquad E_y(t) \rightarrow E_y(\omega)
\end{gathered} \tag{41}
$$

then we have two equations

$$
E_x(\omega) = Z_{xx} H_x(\omega) + Z_{xy} H_y(\omega) \tag{42a}
$$

$$
E_y(\omega) = Z_{yx} H_x(\omega) + Z_{yy} H_y(\omega) \tag{42b}
$$

but four unknowns.

But the tensor elements Z_{ij} change slowly with ω and can be calculated for fewer frequencies than the transformed values of E and H.

The most common way is to multiply (42a) and (42b) with the complex conjugate of the magnetic fields and average over frequency intervals, we get

$$
< E_x H_x^* > = Z_{xx} < H_x H_x^* > + Z_{xy} < H_y H_x^* > \tag{43a}
$$

$$
< E_y H_x^* > = Z_{yx} < H_x H_x^* > + Z_{yy} < H_y H_x^* > \tag{43b}
$$

$$
< E_x H_y^* > = Z_{xx} < H_x H_y^* > + Z_{xy} < H_y H_y^* > \tag{43c}
$$

$$
< E_y H_y^* > = Z_{yx} < H_x H_y^* > + Z_{yy} < H_y H_y^* > \tag{43d}
$$

where

$$< AB^* > (\omega) = \frac{1}{\omega} \int_{\omega - \frac{\Delta\omega}{2}}^{\omega + \frac{\Delta\omega}{2}} A(\omega') B^*(\omega') \, \mathrm{d}\omega' \tag{44}$$

Equations (43a-d) can be solved for Z_{ij}

$$Z_{xx}(\omega) = \frac{1}{D_1} \left[< E_x H_x^* >< H_y H_y^* > - < E_x H_y^* >< H_y H_x^* > \right] \tag{45a}$$

$$Z_{yx}(\omega) = \frac{1}{D_1} \left[< E_y H_x^* >< H_y H_y^* > - < E_y H_y^* >< H_y H_x^* > \right] \tag{45b}$$

$$Z_{xy}(\omega) = \frac{-1}{D_1} \left[< E_x H_x^* >< H_x H_y^* > - < E_x H_y^* >< H_x H_x^* > \right] \tag{45c}$$

$$Z_{yy}(\omega) = \frac{-1}{D_1} \left[< E_y H_x^* >< H_x H_y^* > - < E_y H_y^* >< H_x H_x^* > \right] \tag{45d}$$

where

$$D_1 =< H_x H_x^* >< H_y H_y^* > - < H_x H_y^* >< H_y H_x^* > \tag{46}$$

For one dimensional resistivity structure (horizontal layered earth) we have

$$\bar{\bar{Z}} = \begin{pmatrix} 0 & Z \\ -Z & 0 \end{pmatrix}; \qquad Z = |Z| e^{i\phi}; \qquad \rho_a = \frac{1}{\mu\omega} |Z|^2 \tag{47}$$

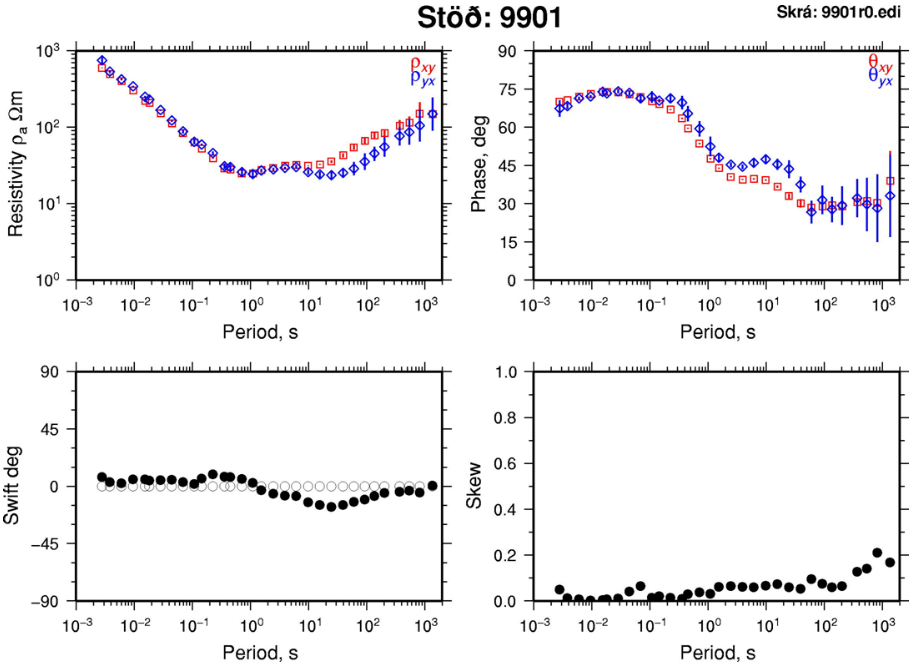

Figure 7: 1D data.

5. 2D EARTH

For two dimensional resistivity structure (Figure 8) it can be shown that if x is along strike then

$$\bar{\bar{Z}} = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix}; \qquad Z_{xy} = |Z_{xy}|e^{i\phi_{xy}}; \qquad Z_{yx} = |Z_{yx}|e^{i\phi_{yx}}; \tag{48}$$

and we have two apparent resistivities and phases (Figure 9)

$$\underbrace{\rho_{xy} = \frac{1}{\mu_0\omega}|Z_{xy}|^2; \qquad \phi_{xy}}_{\text{TE}-\text{mode}} \qquad \underbrace{\rho_{yx} = \frac{1}{\mu_0\omega}|Z_{yx}|^2; \qquad \phi_{yx}}_{\text{TM}-\text{mode}} \tag{49}$$

The strike is not known at forehand and we rotate the actual directions x and y to x' and y'

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\bar{\bar{R}}} \begin{pmatrix} E_x \\ E_y \end{pmatrix}; \quad \begin{pmatrix} H'_x \\ H'_y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\bar{\bar{R}}} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \tag{50}$$

In the rotated coordinate system the impedance tensor is

$$\bar{\bar{Z}}' = \bar{\bar{R}} \cdot \bar{\bar{Z}}\bar{\bar{R}}^{\mathrm{T}} \tag{51}$$

and the rotation to strike-direction is found by determining θ and hence $\bar{\bar{R}}$ so that

$$\bar{\bar{Z}}' = \begin{pmatrix} 0 & Z'_{xy} \\ Z'_{yx} & 0 \end{pmatrix} \tag{52}$$

In the general case of 3−dimensional resistivity structure all tensor elements are different from zero for all orientations of measurement directions. The skewness, defined as

$$S = \frac{|Z_{xx} + Z_{yy}|}{|Z_{xy} - Z_{yx}|} \tag{53}$$

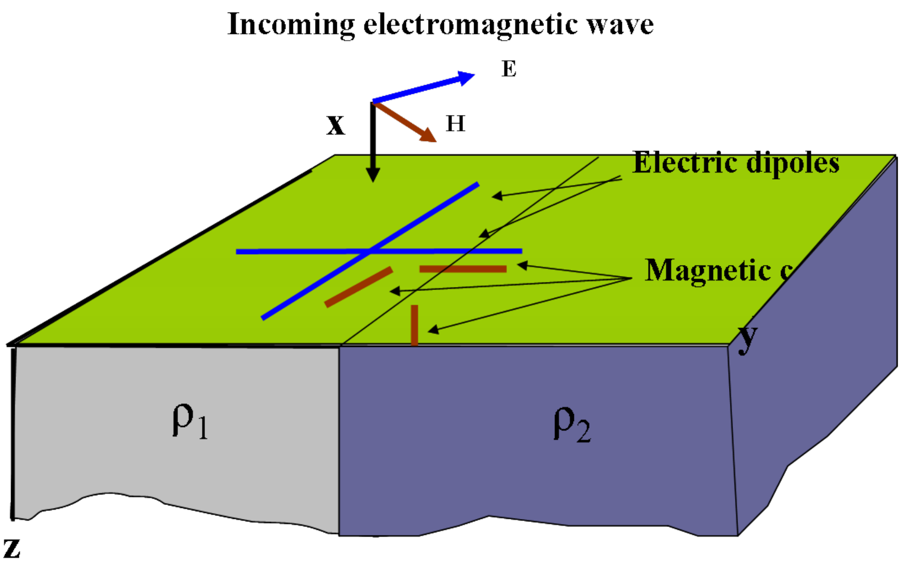

Figure 8: 2D earth.

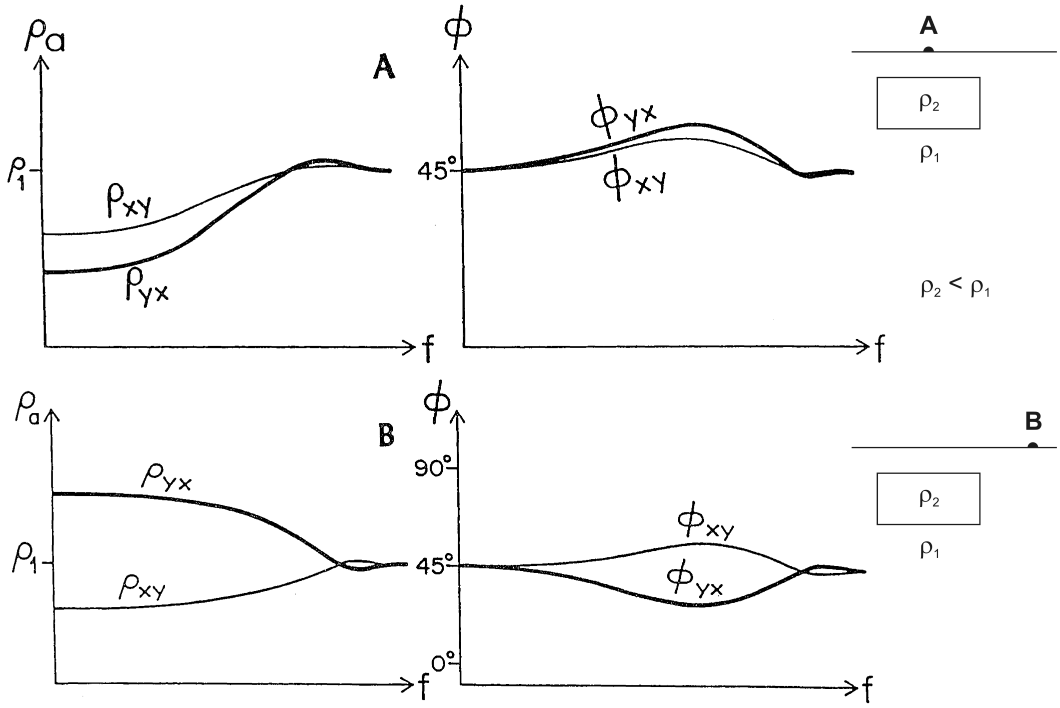

Figure 9: Typical behavior of TE and TM mode apparent resistivities and impedance phases at two points near a 2−D body in a homogeneous half-space

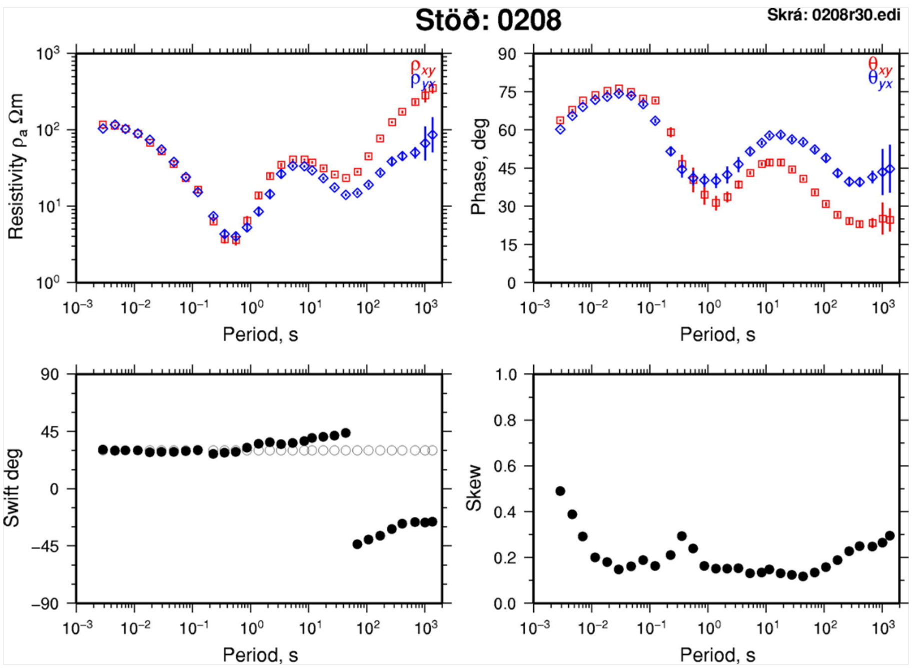

Figure 10: 2D data.

is a measure of how severely the resistivity structure is $3-$dimensional. S is independent of rotation. In $3-$D cases one often uses apparent resistivity based on the determinant of $\bar{\bar{Z}}$ which is independent on rotation

$$\det \bar{\bar{Z}} = Z_{xx}Z_{yy} - Z_{xy}Z_{yx} \tag{54}$$

For 1-D resistivity structure

$$Z_{xx} = Z_{yy} = 0; \qquad Z_{xy} = -Z_{yx} = \hat{Z}_1$$

and

$$\rho_a = \frac{1}{\mu_0 \omega} \left| \det \bar{\bar{Z}} \right| \tag{55}$$

In the $3-$D case this can be used to define a sort of "directional average" apparent resistivity.

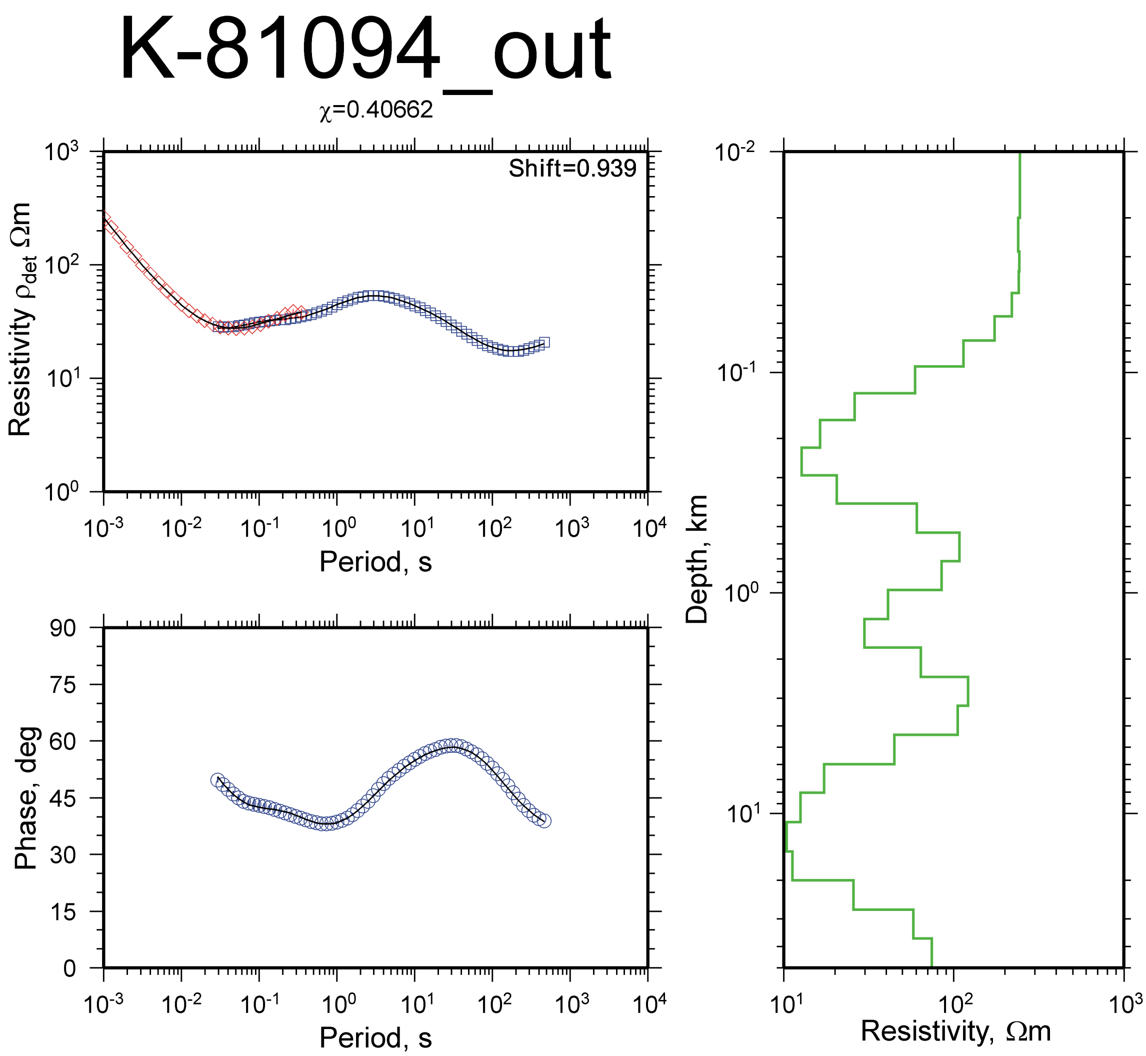

Figure 11: 1D interpretation of Det. apparent resistivity and phase