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SIMULATION OF DISTRICT HEATING IN TIANJIN, CHINA

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ABSTRACT

In order to decrease air pollution and save conventional energy, geothermal energy as a heat source for district heating systems has been used widely in China due to its cleanness and lower operating cost. This paper describes the geothermal resource and district heating system in Tianjin. Heat load for one sample building was calculated using Icelandic heat loss calculations and the Chinese building code. Based on this, both geothermal heating and a fuel-fired system using a steady-state model and a dynamic model were simulated and the results analysed. A sample district heating system network was set up, pressure and temperature drop of pipeline were calculated, and the system simulated. A dynamic simulation model is a powerful tool for a designer to study the system performance and optimise the heating system.

1. INTRODUCTION

A district heating system is composed of many elements, building a chain from the heat source to the heated buildings. The sole purpose of a district heating system is to supply adequate heat to its consumers. The consumer uses the heat to maintain indoor temperature at a reasonably constant level and counter for building heat loss to the surroundings. The most common method of heat generation for district heating in Tianjin is firing of fossil fuel. The heat is distributed to the consumers through a closed loop network, where the hot water is piped to each consumer in the supply network, cooled down by the heat consumer, piped back to the boiler in the return network and re-heated.

Geothermal energy is abundant in Tianjin, it has been developed and used for district heating because of its suitable temperature (70-90°C) in recent years. Compared to the conventional energy, geothermal energy is at a fixed temperature, which usually can not be controlled by the district heating system operator. Tianjin geothermal district heating system consists of two main subsystems, the geothermal pipeline system and the city distribution loop. The geothermal pipeline system transfers the geothermal fluid from wellheads to the pumping stations. In the pumping stations, the energy of the geothermal fluid is transferred to water which is circulated in the city distribution loop using heat exchangers.

1.1 Geothermal field and wells

In Tianjin, there are 11 geothermally anomalous areas that have been found and comprehensively researched so far. The temperature gradient of the geothermal fields is 35-88°C/km in Tianjin. They are covered with Quaternary and Tertiary sediments except for mountainous areas with a thickness of about 100-9000 m. Under the Tertiary sediments, there are Ordovician, Cambrian and Precambrian formations which are the main geothermal reservoirs.

The total area of the 11 geothermal fields in Tianjin is on the order of 2000 km^2 (Figure 1 and Table 1). All of them are typical medium- to low-temperature geothermal resources. The sedimentary geothermal reservoirs are 500-2000 m deep and the temperature ranges from 30 to 78°C, with a maximum temperature of 98°C.

There are currently 239 geothermal wells in Tianjin, including 10 injection wells. All of them for direct use such as district heating and greenhouses because of their low temperature (< 100°C). Table 2 gives basic information on some geothermal wells.

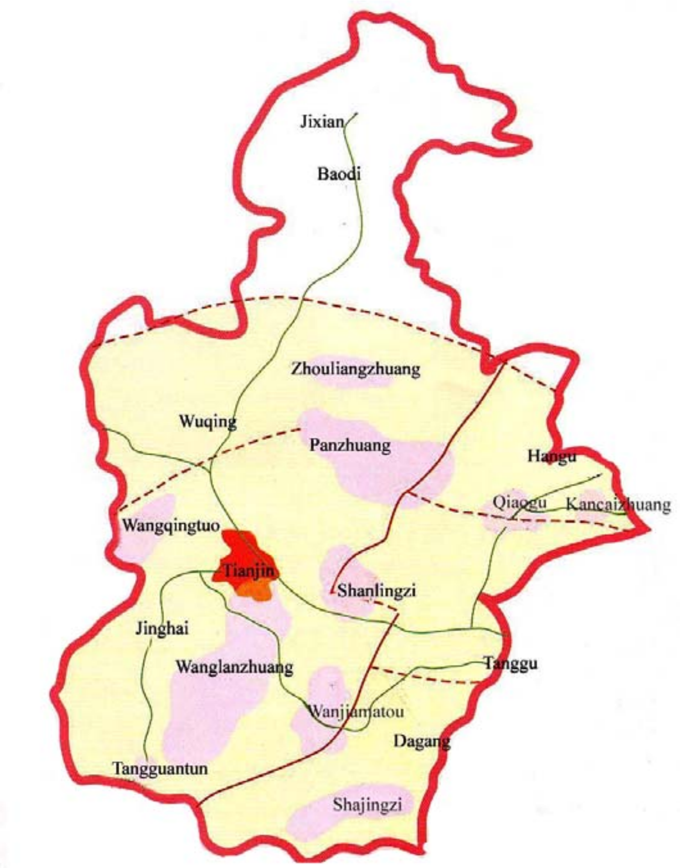

FIGURE 1: Geothermal anomalies in Tianjin

TABLE 1: Basic data on the main geothermal anomalies in Tianjin

Geothermal field	Area (km^2)	Highest gradient (°C /km)
Zhouliangzhuang	200	55
Panzhuang	600	69
Qiaogu	100	55
Wangqingtuo	130	50
Wanglanzhuang	840	88
Wanjiamatou	260	45
Shajingzi	300	76
Tangguantun	60	80

1.2 Geothermal supply network

Tianjin is located in the north temperature zone with an annual mean temperature of 12.3°C, and the average temperature of – 0.9°C during the 135 days of the heating period. The energy consumption of space heating takes 10% of the total. Coal is the major heating energy resource in Tianjin, the others are natural gas, oil products and geothermal energy. Geothermal energy and the three other heating source alternatives are available for district heating applications in Tianjin.

Over almost 20 years, geothermal energy in Tianjin has played an important role in the supply of energy, alleviation of environmental pollution and improvement of resident living. Compared with a

TABLE 2: Basic information on geothermal wells

No.	**Depth (m)**	**Temperature (°C)**	**Mass flow (kg/s)**	**Reservoir**
HX-05	915	56	27.78	Nm
HD-22	1066	49	27.22	Nm
DL-26	1372	51	22.78	Nm
DL-16	1312	70	41.67	O
HX-10	1543	57	22.78	O
DG-08	1878	69	25.00	Ng
TG-02	2222	77.5	36.39	Ng
DL-14	1727	96	57.50	Є
HX-11	2010	92	43.06	Є
HX-25	1608	89	31.11	Jxw
DL-22	2546	94	48.89	Jxw
HD-12	3165	88	43.06	Jxw

conventional heating system using solid fuel, gas fuel or liquid fuel, geothermal energy is very suitable to use as a heat source for district heating. Since its exploration and development, in Tianjin, geothermal energy has quickly attracted significant attention from more and more people despite its late start in the application as a heat source. This is because geothermal energy has the following advantages: compactness of its heat supply station, lower operating cost, high efficiency in its comprehensive utilization, fast in capital cost return, and, especially, its limited contribution to air pollution. In addition, geothermal water has stable heat output, which is beneficial for effective control and regulation of the system. Therefore, using geothermal water as a heating source has the potential for energy saving. To sustain the geothermal energy, in the design of geothermal heating systems, both a production well and an injection well with a particular distance between them needs to be incorporated in the system.

As for a geothermal heating system, it is proven that it can provide a more stable heat source and a more stable indoor temperature compared with conventional heating systems. These benefits have been reflected in the increasing total heating area as well as in the improved efficiency of the district heating system. The main reason is that heat comes from geothermal water 24 hours per day providing a constant and balanced source of energy.

1.2.1 Type of system and pipe

Presently, geothermal heating systems are double systems and FRP (Fibreglass-Reinforced Plastics) pipes are usually used to avoid corrosion. FRP is a compound material composed of colophony and fibreglass. The lifetime of FRP is 10-15 years used for transmitting geothermal water, and now the price is almost the same as that of a steel pipeline. But for the city distribution loop, standard steel pipes are frequently used.

1.2.2 Buildings and building types

The residential buildings in China are normally six stories high. One of the favourable conditions is that the buildings to be heated are concentrated in certain areas and therefore there is a big heat demand relative to the ground area. This means that the heating areas are prone for extensive development.

In principle, heat consumption is based on a function of the building's structure, its insulation characteristics, volume of building and local climate. Following the reference from the Heating Design Handbook (Tang Huifen and Fan Jixian, 1992), design heat consumption per unit area of 46-70 W/m^2 in typical residential buildings is used to calculate heat load of a conventional heating system. The value indicates that heat consumption should be for a period of 147 days, a standard minimum outdoor local temperature of -9ºC, and an indoor temperature of 20ºC. The easy way would be to use the value from the handbook, but a value should be selected with respect to the actual situation. With a detailed approach, the value can be calculated.

1.3 Climate

Air temperature affects the indoor temperature through heat conduction in the external walls, windows and through free and forced infiltration.

In order to obtain good simulation results, climate data should be used. Figure 2 shows the outdoor temperature during one year in Tianjin.

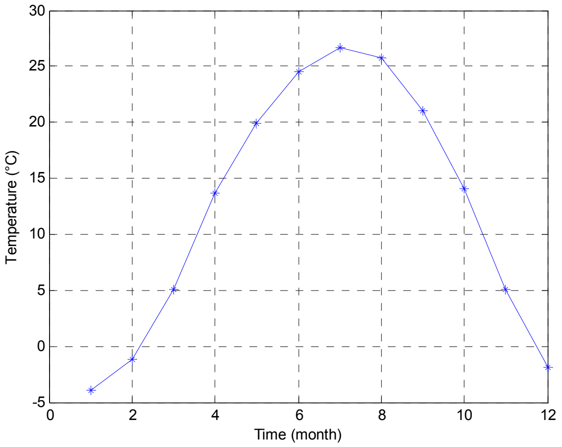

FIGURE 2: Monthly mean temperature in Tianjin

2. BUILDING CALCULATION

The heat load of a building consists of the following components:

1) Heat loss through building surroundings;
2) Heat loss by infiltration;
3) Heat load of inrush air due to opening of the doors and windows heat loss;
4) Radiation through building surrounding the entering room.

In addition, there is other heat load, such as due to illumination, human body and other additional heat load, which is neglected in this report because it is small compared to the above mentioned heat load and loss. Basic theory for heat transfer in buildings is covered in Appendix I.

2.1 Sample building

Figure 3 shows a typical building in Tianjin, which has six stories and four flats of three types (A, B and C) on the same floor. The total area of this building is shown in Table 3.

TABLE 3: Total area of the building

Element	**A (m^2)**	**B (m^2)**	**C (m^2)**	**Total area (m^2)**
Roof	913.76	495.76	415.48	1825.00
Floor	913.76	495.76	415.48	1825.00
External walls	2537.04	2608.32		5145.36
Internal walls	3520.80	5094.00		8614.80
Windows	418.56	444.48		863.04

It is necessary to identify the character of the different construction materials so as to evaluate the building parameters. They are given according to Emeish (2001) in Table 4, and Figure 4 shows examples of their use in the building. The heat transfer coefficients for doors and windows of different materials are given in Table 5.

FIGURE 3: Sample building in Tianjin

TABLE 4: Construction material's properties

Construction material	Density, ρ (kg/m^3)	Thermal conductivity, λ (W/(m °C))	Heat capacity, C (kJ/(kg °C))
Concrete	2088	1.21	1.08
Brick	1800	0.60	1.80
Chalk	2710	4.64	0.86
Asphalt mix	2000	0.70	-
Roof bricks	1400	0.95	1.08
Sand	1450	0.38	0.92
Cement tiles	2145	1.35	0.96

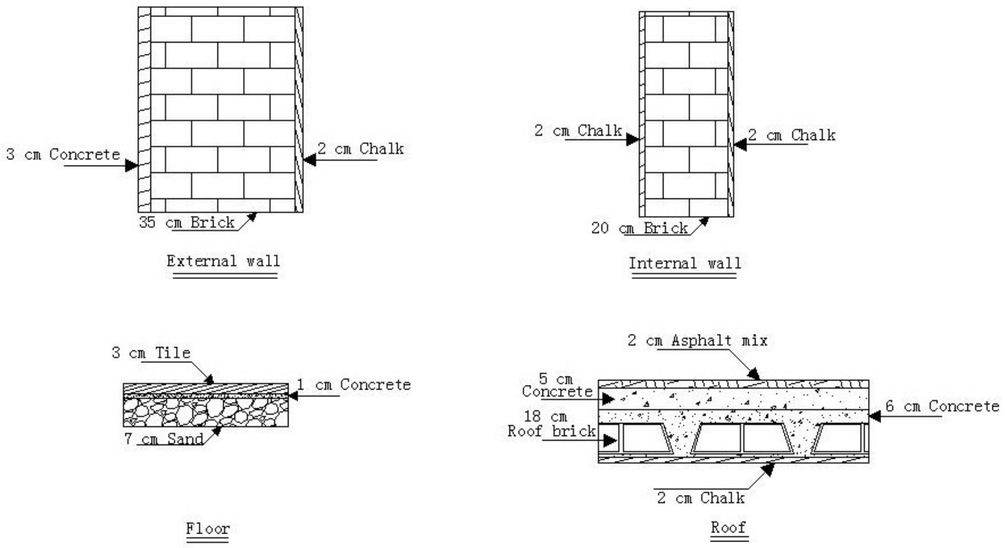

FIGURE 4: The surrounding structure of a building

TABLE 5: Overall heat transfer coefficients (W/(m^2 °C)) for doors and windows

Window or door material	Door	Single glazing	Double glazing
Wood	3.50	9.00	3.00
Steel	5.80	6.70	3.50
Aluminium	7.00	6.70	3.50

2.2 Heat losses through building surroundings

The total heat transfer coefficient and thermal mass are indispensable to the simulation of a district heating system.

2.2.1 Heat transfer coefficient

Heat transfer coefficient, U is a constant that describes the heat transfer between the building and its environment due to conduction, convection and radiation. U_{total} (W/(m^2 °C)) can be calculated according to the following formula:

$$U_{total} = \frac{U_1 A_1 + U_2 A_2 + ...}{A_{total}} \tag{1}$$

where U_n = Heat transfer coefficient of different components of the building (walls, windows, roof and floors) (W/(m^2 °C)); also $U_n = 1/R_n$;
A_n = Surface area of component n (m^2);
A_{total} = Total surface area of the building, (m^2);
R_n = The thermal resistance of heat transfer for component n (m^2 °C/W);
R_{total}= Total thermal resistance of heat transfer (m^2 °C/W).

The heat transfer coefficient will be calculated for this building without insulation.

1) Wall heat transfer coefficient

The walls total transfer coefficient is given in Table 6.

TABLE 6: External walls heat transfer coefficient

Construction material	***x*** (m)	***λ*** (W/(m °C))	***R*** (m^2 °C /W)
Concrete	0.03	1.21	0.02
Brick	0.35	0.60	0.58
Chalk	0.02	4.64	0.01
R_i			0.12
R_o			0.04
R_{total}			0.77

(R_i and R_o are given in Tables 1 and 2 in Appendix I)

$U_{wall} = 1/R_{total} = 1/0.77 = 1.29$ W/(m^2 °C

2) Roof heat transfer coefficient

From Figure 4, it is seen that the construction materials of the roof consist of two kinds of material, concrete and roof brick, and concrete covers 80% of the total area and 20% is of roof brick. So two different heat transfer coefficients need to be calculated. The results are shown in Tables 7 and 8.

TABLE 7: Roof's heat transfer coefficient using brick

Construction material	***X* (m)**	***λ* (W/(m°C))**	***R* (m^2 °C /W)**
Asphalt	0.02	0.7	0.03
Concrete	0.05	1.21	0.04
Reinforced concrete	0.06	1.75	0.03
Roof brick	0.18	0.95	0.19
Chalk	0.02	4.65	0.00
R_i			0.10
R_o			0.04
R_{total}			0.44

$U_1 = 1/R_{total} = 1/0.44 = 2.28$ W/(m^2 °C)

TABLE 8: Roof's heat transfer coefficient without brick

Construction material	***x* (m)**	***λ* (W/(m °C))**	***R* (m^2 °C /W)**
Asphalt	0.02	0.7	0.03
Concrete	0.05	1.21	0.04
Reinforced concrete	0.24	1.75	0.14
Chalk	0.02	4.65	0.00
Ri			0.10
R_o			0.04
R_{total}			0.35

$U_2 = 1/R_{total} = 1/0.35 = 2.85$ W/(m^2 °C)

Hence:

$$U_{roof} = \frac{U_1 A_1 + U_2 A_2}{A_{total}} = 2.40 \text{ W/(m}^2\text{ °C)}$$

3) Floor heat transfer coefficient

Based on Figure 4, the total heat transfer coefficient of the floor is given in Table 9.

TABLE 9: Floor heat transfer coefficient

Construction material	***x* (m)**	***λ* (W/(m °C))**	***R* (m^2 °C /W)**
Sand	0.07	0.38	0.18
Concrete	0.01	1.21	0.01
Tiles	0.03	1.35	0.02
R_i			0.15
R_o			0.09
R_{total}			0.45

$U_{floor} = 1/R_{total} = 1/0.45 = 2.22$ W/(m^2 °C)

4) Windows

As for windows, the type used in Tianjin is single glazing with a *U* value of 9 W/(m^2 °C).

2.2.2 Heat loss in stairs calculation

Calculation on the stairs temperature is, according to Equation 2, based on the energy balance and the area of each component (Table 10).

$$\sum U_i A_i (T_i - T_{stair}) = \sum U_o A_o (T_{stair} - T_o) \qquad (2)$$

TABLE 10: Stairs heat transfer coefficient

Type	***A* (m^2)**	***U* (W/(m^2 °C))**
Internal wall	333.00	1.21
External wall	95.40	1.31
Doors in internal wall	36.00	3.5
Windows in external wall	60.00	9

Equation 2 gives T_{stair} = 4.0°C

The heat loss from the stairs is: $Q_s = 8.54$ kW

2.2.3 Building total heat transfer coefficient

On the basis of the above calculations, the total heat transfer coefficient of a building can be summarized as done in Table 11.

TABLE 11: Total heat transfer coefficient of a building

Element	***U* (W/(m^2 °C))**	***A* (m^2)**	***UA* (W/°C)**
Roof	2.40	1825.00	4372.57
Floor	2.22	1825.00	4051.50
External walls	1.31	5145.36	6748.65
Windows	9.00	863.04	7767.36
Total			22,940

2.3 Building thermal mass

The thermal mass is the capacity of a building to store heat. It is the second parameter needed for simulation. There is a temperature gradient when heat is transferred through multi-layer walls, with the temperature of the layer adjacent to the outside low. The thermal mass can be defined as the part of a wall that can store heat to maintain the room temperature at 20°C. Surface temperature of every layer (Figure 5) can be calculated according to Equation 3:

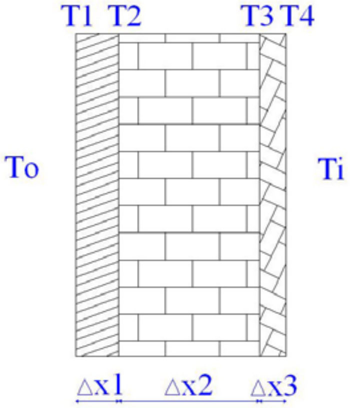

FIGURE 5: A sample of the external wall

$$Q = \frac{T_o - T_1}{R_o} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_i}{R_i} \qquad (3)$$

Solving Equation 3, with T_i = 20°C, T_o = -9°C and Q = 38 W, yields the following:

T_1 = -7.9°C, T_2 = -6.9°C, T_3 = 15.3°C, T_4 = 15.4°C.

1) External walls thermal mass

The thermal mass of the external walls which consist of bricks and chalk is given in Table 12:

TABLE 12: External walls thermal mass

Construction material	***x*** **(m)**	***A*** **(m^2)**	***ρ*** **(kg/m^3)**	***c*** **(kJ/(kg °C))**	***C*** **(kJ/°C)**
Brick	0.2	5145.36	1380	1.08	1533728.91
Chalk	0.03	5145.36	2710	0.86	359753.28
C_{total}					1893482

2) Floor thermal mass

The thermal mass of the floor which consists of concrete and tiles is given in Table 13.

TABLE 13: Floor thermal mass

Construction material	***x*** **(m)**	***A*** **(m^2)**	***ρ*** **(kg/m^3)**	**c (kJ/(kg °C))**	***C*** **(kJ/°C)**
concrete	0.01	1825.00	20880.00	1.08	411544.80
Tiles	0.03	1825.00	2145.00	0.96	93951
C_{total}					505496

3) Roof thermal mass

As for the roof, its thermal mass is shown in Tables 14 and 15 for the bricks and concrete parts, respectively.

TABLE 14: Roof thermal mass (with bricks)

Construction material	***x*** **(m)**	***A*** **(m^2)**	***ρ*** **(kg/m^3)**	***c*** **(kJ/kg °C)**	***C*** **(kJ/°C)**
Roof brick	0.18	1460	1400	1.08	397353.6
Chalk	0.02	1460	2710	0.86	68053.52
C_{total}					465407

TABLE 15: Roof thermal mass (without bricks)

Construction material	***x*** **(m)**	***A*** **(m^2)**	***ρ*** **(kg/m^3)**	***c*** **(kJ/(kg °C))**	***C*** **(kJ/°C)**
Concrete	0.24	365.00	2088.00	1.08	197541.50
Chalk	0.02	365.00	2710.00	0.86	17013.38
C_{total}					214555

4) Internal walls thermal mass

The construction and thermal mass of the internal walls is given in Table 16.

TABLE 16: Internal walls thermal mass (brick walls)

Construction material	***x*** **(m)**	***A*** **(m^2)**	***ρ*** **(kg/m^3)**	***c*** **(kJ/(kg °C))**	***C*** **(kJ/°C)**
Chalk	0.02	8614.80	2710.00	0.86	301164.79
Brick	0.20	8614.80	1380.00	1.08	2567899.58
Chalk	0.02	8614.80	2710.00	0.86	301164.79
C_{total}					3170229

5) *Total thermal mass*
Table 17 shows finally the summary of the total thermal mass of the sample building (uninsulated).

TABLE 17: Building's total thermal mass

Surface	C (kJ/°C)
Roof	679962
Floor	505496
External walls	1893482
Internal walls	3170229
C_{total}	6249169

2.4 Heat loss by infiltration

Infiltration is the leakage of outside air into the house through cracks around the windows and doors. The amount of the infiltration depends mainly on the tightness of windows and doors and on the outside wind velocity or the pressure difference between the outside and inside. Two methods can be used to estimate the volume of infiltration air into the heating area.

1) Crack length method
The base of this method is the perimeter of windows and doors, according to the following equation:

$$V = \sum l\, L\, n \quad (\mathrm{m^3/h}) \tag{4}$$

where l = The total length of crack (m);
L = Infiltration of air volume through windows and doors per metre ($\mathrm{m^3/(m\ h)}$);
n = Correction factor of infiltration for different direction.

2) Air change method
This method is based on the air volume in a space for being replaced by outside air certain times per hour. This method is used here for calculating the infiltration. The China heating code recommends 0.5-1.0 air changes per hour for residential buildings. One air change per hour is used in this paper and can be calculated in kJ/h according to:

$$Q_2 = c_p V \rho_d (T_i - T_o) \tag{5}$$

where c_p = Specific heat of air, 1.0056 kJ/kg;
V = Volume flow of infiltrated air, $\mathrm{m^3/h}$;
ρ_d = Air density at the temperature T_o, 1.32 $\mathrm{kg/m^3}$;

Using the air change method to determine V ($\mathrm{m^3/h}$), gives:

$$V = n V_h$$

where n = Air change times per hour, 0.5-1.0/h;
V_h = Room volume, which is 32,850 $\mathrm{m^3}$.

From Equation 5: $Q_2 = 352.26$ kW

Thus, the total heat loss is: $Q_{total} = Q_1 + Q_2 = 1042$ kW

2.5 Building calculation

The following calculations are according to Tang Huifen and Fan Jixian (1992) and are given here for comparison.

2.5.1 Heat load through building surroundings

For steady-state heat transfer, the formula is as follows:

$$Q_1 = UA(T_i - T_o)a \tag{6}$$

where Q_1 = Heat loss through building surrounding (W);
U = Heat transfer coefficient through building surrounding (W/(m^2 °C));
A = Areas of building surrounding, m^2;
T_i = Indoor temperature and outside temperature (°C);
T_o = Outdoor temperature (°C).

TABLE 18: Heat transfer coefficients of each component of building surroundings

Type	***U*** **(W/(m^2 °C))**	***A*** **(m^2)**	***U A*** **(W/°C)**
Roof	0.91	1825	1660.75
Floor	0.3	1825	547.5
External walls	1.24	5145.36	6380.25
Windows	6.4	863.04	5523.46
Stairs	1.83	240	439.2
Doors	2.91	288	838.08
Mean heat transfer coefficient	1.82		

$Q_1 = \sum K_i A_i (T_i - T_o)$ = 446287.74 W = 446.29 kW

2.5.2 Infiltration

From Equation 5: $Q_2 = 351.26$ kW

2.5.3 Heat load of inrush air due to opening of doors and windows

Heat loss per story of a building from external doors and windows is $Q = 24.30$ kW

The additional heat load of inrush air from doors and windows is calculated as:

$Q_3 = N\,65\% \times 24.3 = 94.77$ kW

where N is the number of stories.

2.5.4 Total heat load

According to the handbook (Tang Huifen and Fan Jixian, 1992), if the height of a building is more than 4 m, 2% of the total heat loss should be added for every metre. Q_4 is therefore calculated as:

$Q_4 = 223.31$ kW

This gives the total heat load as:

$$Q_{total} = Q_1 + Q_t + Q_3 + Q_4 = 1116.24 \text{ kW}$$

3. BUILDING HEAT LOAD MODELS

3.1 General

A district heating system consists of buildings, pipes, pump station and heat producing station. Heat can be produced from a geothermal field or by combustion. In this chapter, district heating models are studied. Geothermal district heating systems are compared to existing fuel fired systems. The models which include a steady-state model and dynamic model are programmed with MatLab. The signals used in the district heating systems are summarized in Table 19.

TABLE 19: Main influencing signals in the simulation of district heating systems

Input signals	Control signals	State signals	Output signals
Outdoor temperature	System water supply temp.	Indoor temperature Water quantity in storage	Water flow Return temperature System heat load

3.2 Weather data

Figure 6 shows the outdoor temperature duration curve (Y axis is inverted). It can be seen that outdoor temperature is below 15°C for 3600 hours, or about 5 months. The data is recorded at a resolution of 1°C.

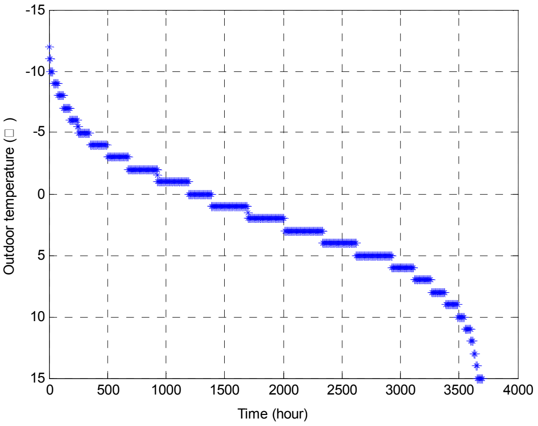

FIGURE 6: Duration curve of outdoor temperature

3.3 Models

Models of district heating systems can be classified as follows:

- By type: microscopic or macroscopic;
- By method: dynamic or steady-state;
- By approach: physical or black box;
- By usage: design or operation.

The concepts "microscopic" and "macroscopic" refer to if the state of the district heating system is to be studied in detail both in time and space, or if the district heating system is lumped into a few model blocks, ignoring spatial variance of the system state. Dynamic models depend on previous state history, whereas steady-state models are time independent and assume steady-state conditions.
In this section, models for a district heating network are described. The models treated are macroscopic and physical. The district heating network is lumped into one model block. These models can be found in Valdimarsson (1993). Variables used in the model theory are defined in Nomenclature.

3.3.1 Radiators

Radiator is the heat exchanger that transfers heat from the heating system to the room air. According to

Anon (1977), the relative heat load of a radiator can be written as:

$$\frac{Q}{Q_0}=\left(\frac{\Delta T_m}{\Delta T_{m0}}\right)^{(4/3)}=\left(\frac{T_s-T_r}{\ln\left(\dfrac{T_s-T_i}{T_r-T_i}\right)}\cdot\frac{\ln\left(\dfrac{T_{s0}-T_{i0}}{T_{r0}-T_{i0}}\right)}{T_{s0}-T_{r0}}\right)^{(4/3)} \tag{7}$$

where Q/Q_o = The ratio of the actual heat output from the radiator to the heat output at design conditions;
T_s = Water supply temperature (°C);
T_r = Water return temperature (°C);
T_i = Indoor temperature (°C).

Supply water temperature is assumed to be around 80°C and the return water temperature is 40°C for geothermal systems. For fuel fired systems, similar values are 90/70°C, with indoor temperature at 20°C. In order to determine the radiator size, the following Equation is used.

The logarithmic mean temperature difference for a radiator, ΔT_m (°C) is defined as:

$$\Delta T_m=\frac{(T_s-T_i)-(T_r-T_i)}{\ln\dfrac{T_s-T_i}{T_r-T_i}} \tag{8}$$

3.3.2 Water heat duty

The heat load, Q (W) due to hot water going through the radiator is:

$$Q=C_p m(T_s-T_r) \tag{9}$$

The relative heat load of the water flow can be written as:

$$\frac{Q}{Q_0}=\frac{m(T_s-T_r)}{m_0(T_{s0}-T_{r0})} \tag{10}$$

3.3.3 Building heat loss

The heat loss of the building can be defined as:

$$Q_{loss}=k_l(T_i-T_o) \tag{11}$$

where k_l = The building heat loss factor, which is a constant.

Relative heat loss is obtained by:

$$\frac{Q_{loss}}{Q_{loss0}}=\frac{T_i-T_o}{T_{io}-T_{oo}} \tag{12}$$

3.3.4 Pipe heat loss

There is heat loss in the pipes from the pumping station to the buildings, which is calculated by using the pipe transmission effectiveness parameter. According to Valdimarsson (1993), the transmission effectiveness, τ is defined as follows:

$$\tau = \frac{T_s - T_g}{T_1 - T_g} = e^{\frac{U_p}{mC_p}} \tag{13}$$

The reference value τ_o can be concluded from the reference conditions:

$$\tau_o = \frac{T_{so} - T_g}{T_{1_o} - T_g} = e^{\frac{U_p}{m_o C_p}} \tag{14}$$

Parameters U_p and C_p are assumed to be constant in the system. Combining Equations 13 and 14, the transmission effectiveness, τ is obtained by:

$$\tau = \tau_o^{\frac{m_m}{m}} \tag{15}$$

Combining Equations 13 and 15, the supply temperature to the building is calculated by:

$$T_s = T_g + (T_1 - T_g)\tau = T_g + (T_1 - T_g)\tau_o^{\frac{m_o}{m}} \tag{16}$$

The return water temperature at the pumping station is obtained from Equation 17:

$$T_2 = T_g + (T_r - T_g)\tau = T_g + (T_r - T_g)\tau_o^{\frac{m_o}{m}} \tag{17}$$

3.3.5 Building energy storage

Buildings will not cool immediately when the heating is stopped because of their heat capacity. The building energy storage model is:

$$\frac{dT_i}{dt} = \frac{1}{C} Q_{net} = \frac{1}{C}(Q_{supp} - Q_{loss}) = \frac{1}{C}(mC_p(T_s - T_r) - k_l(T_i - T_o)) \tag{18}$$

In the steady state model, all time derivatives are set to zero, so this Equation is only used in the dynamic model.

3.4 Steady-state approach

In the steady-state model, buildings are assumed to be with no heat accumulation. Return temperature is calculated by combining Equations 10 and 12 (Nappa, 2000):

$$\frac{Q}{Q_o} = \left(\frac{T_s - T_r}{T_{so} - T_{ro}} . \frac{\ln\left(\frac{T_{so} - T_{io}}{T_{ro} - T_{io}}\right)}{\ln\left(\frac{T_s - T_i}{T_r - T_i}\right)} \right)^{4/3} = \frac{T_i - T_o}{T_{io} - T_{oo}} \tag{19}$$

T_r can be calculated with iteration from Equation 19. According to Valdimarsson (1993), the fastest convergence is obtained, when T_r inside logarithm is calculated with:

$$T_{r,n+1} = (T_s - T_i)e^{-z} + T_i \tag{20}$$

where z is:

$$z = \frac{T_s - T_{r,n}}{T_{so} - T_{ro}} \left(\frac{T_{io} - T_{oo}}{T_i - T_o} \right)^{3/4} \ln\left(\frac{T_{so} - T_{io}}{T_{ro} - T_{io}} \right)$$

With the outside temperature given, T_i is assumed to be constant and the supply temperature is known. Hence $T_{r,n+1}$ can be found iteratively from Equation 20.

In the steady-state model, the heat loss from buildings is the same as the heat load supply, i.e.:

$$Q_{supp} = Q_{loss} \tag{21}$$

or

$$mC_p(T_s - T_r) = k_l(T_i - T_o) \tag{22}$$

Mass flow is obtained directly from Equation 22:

$$m = \frac{k_l(T_i - T_o)}{C_p(T_s - T_r)} \tag{23}$$

The constant k_l can be calculated from the reference conditions:

$$k_l = \frac{m_o C_p(T_{so} - T_{ro})}{T_{io} - T_{oo}} \tag{24}$$

3.5 Dynamic approach

3.5.1 Return temperature calculation

In the steady-state model, T_r was found by combining Equations 10 and 12. This is not a valid approach in dynamic simulations due to energy stored in the buildings. So T_r should be calculated from Equation 10 by an iteration loop:

$$T_{r,n+1} = (T_s - T_i)e^{-y} + T_i \tag{25}$$

where

$$y = \left(\frac{T_s - T_{r,n}}{T_{so} - T_{ro}} \right)^{(-1/4)} \left(\frac{m}{m_o} \right)^{(-3/4)} \ln\left(\frac{T_{so} - T_{io}}{T_{ro} - T_{io}} \right)$$

3.5.2 Relation between mass flow and indoor temperature

The flow controller in the system is unknown. There is no simple physical relation between water flow and indoor temperature, and different buildings have different regulation systems. Each consumer has his own preferences about the indoor temperature and how to change it. The relation between the indoor temperature and the water flow has to be presented as some average of all consumers in the system. Here P-control (proportional) is used.

The P-controller is presented by Equation 26:

$$m = k_p (T_{i\_set} - T_i) + m_{ave} \tag{26}$$

By differentiation of Equation 26 we get:

$$\frac{dm}{dt} = k_p . \frac{dT_i}{dt} \tag{27}$$

T_i can be solved from Equation 26, as:

$$T_i = T_{i\_set} - \frac{m - m_{ave}}{k_p} \tag{28}$$

Equation 18 can be written as follows:

$$\frac{dT_i}{dt} = -\frac{k_l}{C} T_i + \frac{C_p}{C}(T_s - T_r)m + \frac{k_l}{C} T_o \tag{29}$$

Combining Equations 27, 28 and 29, gives Equation 30:

$$\frac{dm}{dt} = -\frac{k_p}{C}\left(C_p (T_s - T_r) + \frac{k_l}{k_p} \right) m - \frac{k_l k_p}{C} T_o + \frac{k_p k_l}{C} (T_{i-set} + \frac{m_{ave}}{k_p}) \tag{30}$$

Equation 30 can also be written in a matrix form as:

$$\left[\frac{dm}{dt} \right] = \left[-\frac{k_p}{C}\left(C_p (T_s - T_r) + \frac{k_l}{k_p} \right) \right] m + \left[-\frac{k_l k_p}{C} \quad \frac{k_l k_p}{C}\left(T_{i-set} + \frac{m_{ave}}{k_p} \right) \right] \begin{bmatrix} T_o \\ 1 \end{bmatrix} \tag{31}$$

3.6 Heat exchangers

Heat exchangers transfer heat, Q from one flow stream to another, without mixing the fluids. They are thus elements with four connection points, as shown in the schematic (Figure 7).

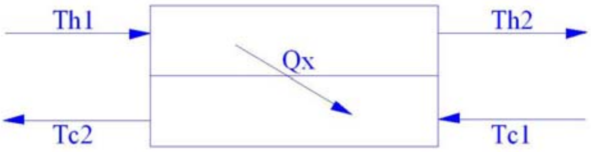

FIGURE 7: Schematic of a heat exchanger

Here:

$$Q = m_h cp (T_{h1} - T_{h2}) \tag{32}$$

$$Q = m_c cp (T_{c1} - T_{c2}) \tag{33}$$

$$Q = UA \Delta T_m \tag{34}$$

and

$$\Delta T_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln \dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}} \tag{35}$$

In order to model the heat exchanger within the network, an equivalent model with two connection points has to be introduced. The equivalent heat transfer coefficient is associated with this simplification. This coefficient is non-linear and dependent on the fluid temperatures, so iteration is necessary for an exact thermal solution.

The heat flow for the heat exchanger elements is then calculated by:

$$Q = UA(T_{h1} - T_{c2}) = UA(T_{h2} - T_{c1})$$

Now it is assumed that there is a 5°C difference, with U = 3000 W/m^2, Q = 1042 kW, T_{h1} = 88°C, T_{h2} = 45°C, T_{c1} = 40°C, T_{h2} = 45°C.

Hence, the heat exchanger area is calculated as: A = 69.46 m^2.

3.7 Reference values and constants

All reference values are marked with subscript *o*. Common reference values for a geothermal district heating network are:

1) Supply water temperature T_{so} = 80°C;
2) Return water temperature T_{ro} = 40°C;
3) Indoor temperature T_{io} = 20°C.

Common reference values used for a fuel fired network are:

1) Supply water temperature for network T_{so} = 90°C
2) Return water temperature for network T_{ro} = 70°C
3) Indoor temperature T_{io} = 20°C

The reference outside temperature depends on the local climate. The reference value used for Tianjin is T_{oo} =–9°C and was used here both for the geothermal and the fuel fired systems. Ground temperature was assumed to be constant at 14.2°C. The reference mass flow of water is related to the size of the network to be studied. Here it was selected to be 5 kg/s. The specific heat capacity of water assumed to be constant, which is, C_p = 4.186 kJ/(kg °C). The P-control parameter kp = 4.5 kg/(s °C). Finally, the heat capacity of the buildings is calculated as 6,249,169 kJ/°C.

4. SIMULATION AND NETWORK CALCULATION

The models described above were programmed with Matlab, and some figures were plotted and analysed for the steady-state model and dynamic model, respectively.

4.1 Simulation results

4.1.1 Steady-state modelling

Results analysis

1) Figures 8 and 9 show the duration curve of supply temperature, return temperature, mass flow and outdoor temperature during the heating period (5 months). From Figure 2 it can be seen that if the maximum mass flow is limited to 3 kg/s, indoor temperature would be less than 20°C for 21 days. The indoor temperature can be regulated by controlling the mass flow to mend this bad condition.

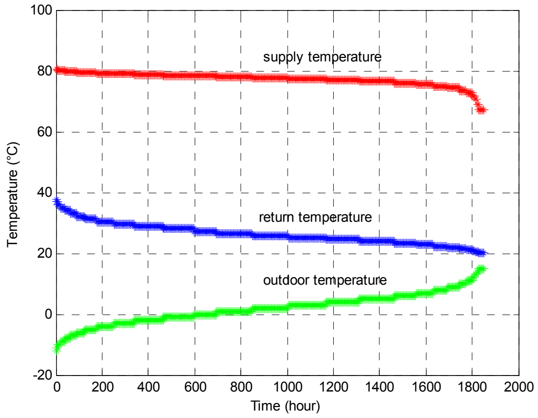

FIGURE 8: Duration curve of temperatures

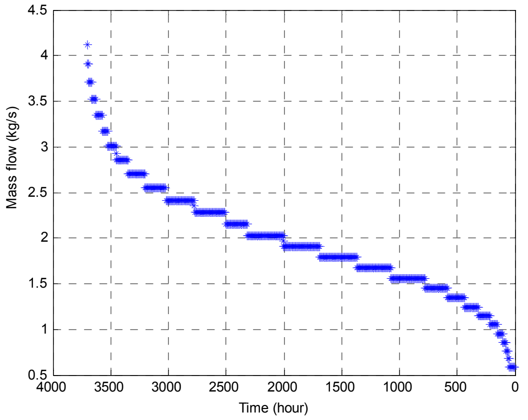

FIGURE 9: Duration curve of mass flow

2) Figure 10 shows the relationship between return temperature and mass flow. It indicates that if the mass flow is higher, the return temperature will be higher. This is because heat load is constant under steady-state conditions. When the mass flow is increased, the temperature difference between supply and return temperature will be smaller, the return temperature increases accordingly.

3) Figure 11 shows the temperature in the different locations of the network. There is temperature difference between T_1 and T_s. This is because there is heat loss in the pipes, and this temperature difference becomes bigger with distance from the source. Also it can be seen that T_r is decreasing as T_o increases. This is because less heat load is needed when T_o increases, so the logarithmic temperature difference is reduced.

4) Figure 12 shows the dependence of mass flow on outdoor temperature for the two kinds of district heating systems. The mass flow of the geothermal system is less than that of the fuel-fired system, which is because the logarithmic temperature difference of the former is smaller than that of the latter (see Section 3.7). The supply/return temperature in the geothermal heating system and the fuel fired system are 80/40°C and 90/70°C, respectively. Therefore, for the same building, it can be concluded that the radiator area of a geothermal system is larger than that of a fuel-fired system. Accordingly, the system cost is higher.

5) Figure 13 shows the relationship between pipe transmission effectiveness and outdoor temperature. Similar to Figure 7, it can be seen that when the relative mass flow is higher, the heat loss in the pipe will decrease, and thus the pipe transmission effectiveness will increase.

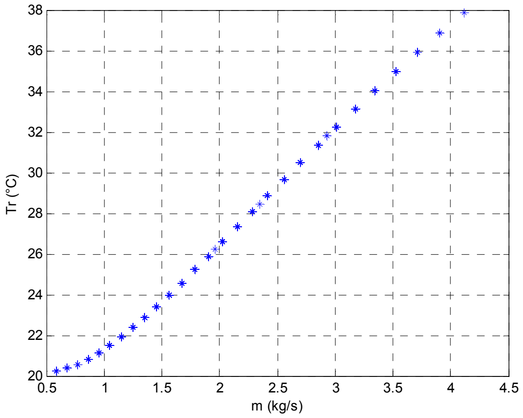

FIGURE 10: Return temperature as a function of mass flow for a geothermal system

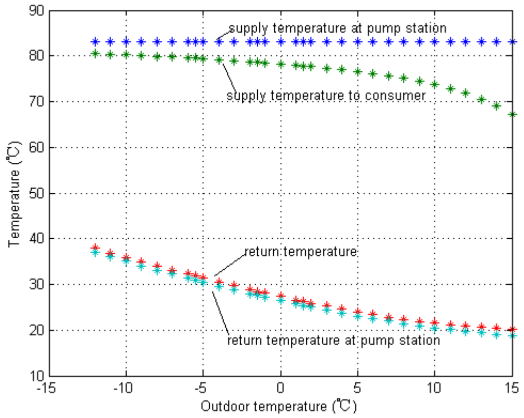

FIGURE 11: Temperature in a geothermal heating system, heat loss in pipe included

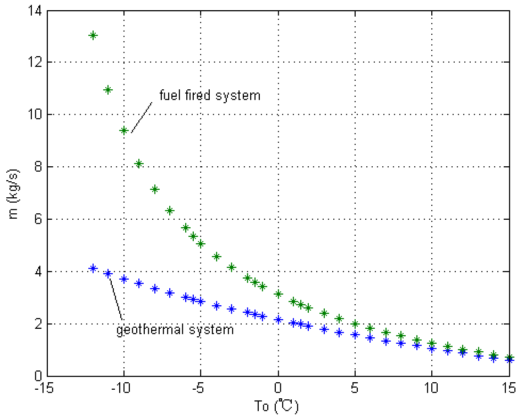

FIGURE 12: Comparison of mass flow of geothermal system and fuel fired system

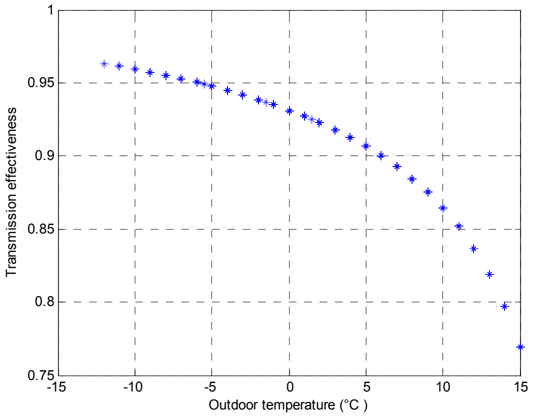

FIGURE 13: The pipe transmission effectiveness with outdoor temperature

4.1.2 Dynamic modelling

In dynamic modelling, initial values for the mass flow m and for the indoor temperature T_i were guessed. The indoor temperature in the steady-state model is constant, but in the dynamic model it is calculated by the building cooling differential equation. Figures 14-17 show some results.

Results analysis

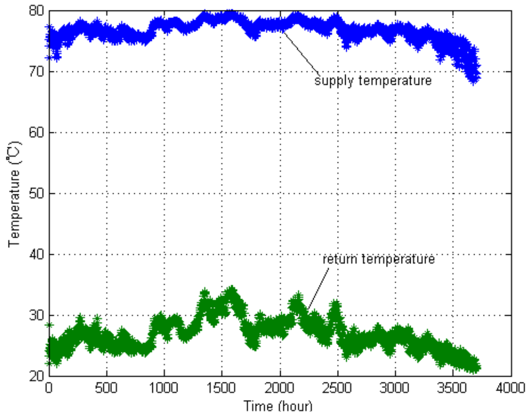

FIGURE 14: Supply temperature and return temperature for the dynamic model

1) Figure 14 shows the supply temperature and return temperature without mass flow limitation. The temperature difference between supply and return temperature is considerable, the average temperature difference is about 48°C. Therefore, for the existing radiator, return temperature should be reduced as possible in order to maintain the desired indoor temperature.

2) Figure 15 shows the relationship between supply temperature, return temperature and outdoor temperature without mass flow limitation. It can be seen that when outdoor

temperature is increasing, supply temperature will decrease, and return temperature is decreasing even here. This is because there is small heat needed with outdoor temperature increasing, and the radiator logarithmic temperature difference decreases.

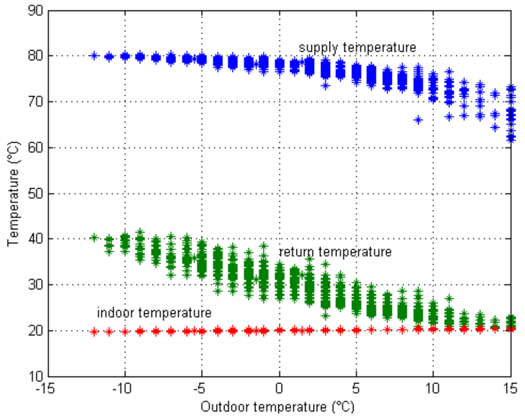

FIGURE 15: Supply, return and indoor temperature as a function of outdoor temperature

3) Figure 16 shows the duration curve of indoor temperature for different radiator sizes. In Figure 16, k_p = 25 kg/s °C, $m_{\_max}$ = 5 kg/s, and for the original radiator size, there are 400 hours during the heating period that the indoor temperature is less than 19.5°C. If radiator size is doubled, indoor temperature conditions are better. So when the maximum mass flow is limited, increasing the radiator size can reduce drop-in indoor temperature significantly. Also when the maximum mass flow is changed, the indoor temperature changes correspondingly, so radiator size is an important parameter for the district heating system. Table 20 shows the return temperature corresponding to the different radiator sizes, at design condition.

4) Figure 17 shows the comparison of different maximum mass flow for different radiator sizes. The maximum flows are 2.5 kg/s and 5 kg/s, and the radiator sizes are 1, 2 and 3 times, respectively. From Figure 17, it can be seen that when the maximum mass flow is 2.5 kg/s, indoor temperature condition is bad despite increasing the radiator size, so this mass flow is too low. For the 5 kg/s maximum mass flow, 2 times radiator size is better than 1 time (baseline), therefore this is the preferable selection for the heating system.

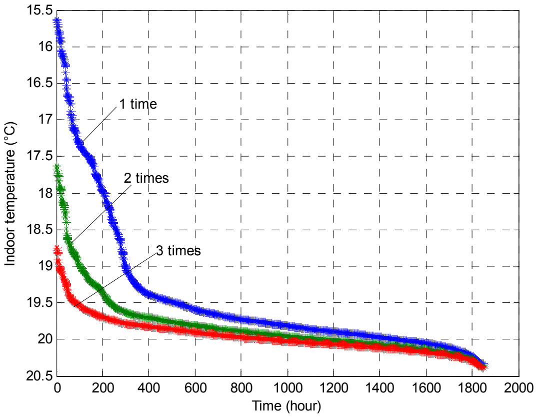

FIGURE 16: Duration curve of indoor temperature for different maximum mass flow and radiator sizes

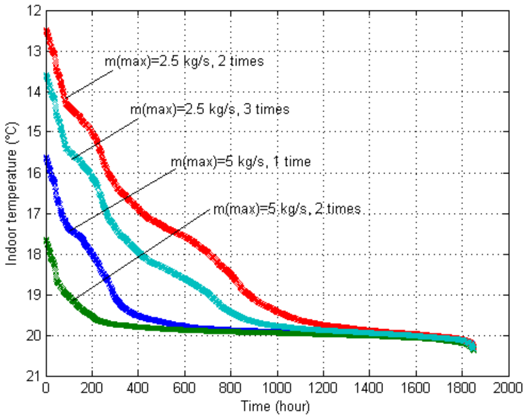

FIGURE 17: Comparison of different maximum mass flow for different radiator sizes

TABLE 20: Influence of radiator size on return temperature and mass flow

Radiator size times	***Tr* (°C)**	***m* (kg/s)**
2	59.8	8.2
3	52.5	6.6
4	47.0	5.8

4.1.3 Radiator selection

Based on the heat output of the radiators, suitable radiator sizes are given in Table 21.

TABLE 21: Radiator selection

Type	**Area (m^2)**	**Heat load (kW)**	**Heat load for one room (W)**	**Output (W)**	**Length (m)**	**Type**
A	138.14	10.87	2718	2805	2	22-60-C
B	149.24	11.75	2936	3087	1.6	33-60-E
C	125.92	9.91	2477	2524	1.8	22-60-C

4.2 Network calculation

As stated in Section 3.3, so-called microscopic models can be used to describe the spatially distributed district heating system behaviour. The goal of developing such models is to be able to calculate the water flow, pressure and temperature in all pipes of the system as a function of time. Network theory provides convenient ways of determining the flow in a given network. The thermal state of the network can then be calculated from the flow solution. According to Valdimarsson (2001), for modelling of district heating network these basic laws have to be fulfilled:

- Conservation of mass;
- Conservation of momentum;
- Conservation of energy.

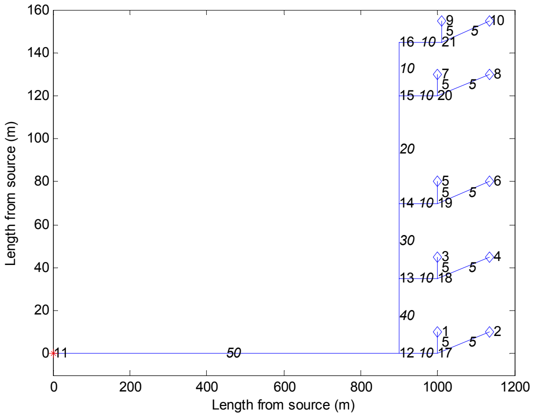

FIGURE 18: Scheme of sample district heating system network

A simple district heating system, containing typical elements for such a system is shown in Figure 18. Numbers 1-10 are node numbers. Between the node numbers are mass flow values in corresponding pipes. More detailed models of networks can be found from Frederiksen (1982).

4.2.1 Pipe

The pipes have a resistance defined by the Darcy-Weisbach equation, which is written as:

$$h = \frac{v^2}{2g}\frac{L}{D}f = \frac{8m^2Lf}{D^2\rho^2\pi^2 g} \tag{37}$$

The friction factor can be calculated from the Colebrook - White equation:

$$\frac{1}{\sqrt{f}} = \left(\frac{a}{\mathrm{Re}\sqrt{f}} + \frac{b}{kD}\right)^2 \tag{38}$$

Table 22 shows pipes used in the system and their properties.

4.2.2 Cost functions

The most common cost function is the monetary function, where investment and operating cost for the system are added. The investment cost increases with increasing pipe diameter, but the operating cost falls with increasing diameter (at least the pumping cost). Pipe price calculation is shown in Table 22. The district heating practice is to design for about 1 bar/km pressure loss.

TABLE 22: Pipe calculation

Name	**node**	**node**	**L (m)**	**m (kg/s)**	**Std. dev. (m)**	**Cost (EUR/m)**	**P (m)**	**T (°C)**	**Total heat loss (W)**	**Total price EUR**
DN200	11	12	900	50	0.2101	69.31	93.32	79.80	20924.59	62375.24
DN150	12	13	35	40	0.1603	52.53	92.22	79.68	31.61	1838.69
DN150	13	14	35	30	0.1603	52.53	92.77	79.79	31.86	1838.69
DN125	14	15	50	20	0.1325	43.39	91.67	79.67	63.72	2169.30
DN100	15	16	25	10	0.1071	37.29	92.44	79.78	14.62	932.18
DN100	12	17	100	10	0.1071	37.29	91.35	79.66	234.02	3728.71
DN100	13	18	100	10	0.1071	37.29	91.89	79.77	234.01	3728.71
DN100	14	19	100	10	0.1071	37.29	90.80	79.65	233.99	3728.71
DN100	15	20	100	10	0.1071	37.29	91.59	79.75	233.95	3728.71
DN100	16	21	110	10	0.1071	29.66	90.58	79.64	283.03	3262.50
DN80	17	1	10	5	0.0825	29.66	94.27	79.86	2.30	296.59
DN80	17	2	136	5	0.0825	29.66	93.73	79.86	424.78	4033.63
DN80	18	3	10	5	0.0825	29.66	93.40	79.85	2.30	296.59
DN80	18	4	136	5	0.0825	29.66	92.85	79.83	424.76	4033.63
DN80	19	5	10	5	0.0825	29.66	92.63	79.82	2.30	296.59
DN80	19	6	136	5	0.0825	29.66	93.40	79.81	424.72	4033.63
DN80	20	7	10	5	0.0825	29.66	92.85	79.80	2.30	296.59
DN80	20	8	136	5	0.0825	29.66	92.53	79.79	424.65	4033.63
DN80	21	9	10	5	0.0825	29.66	91.98	79.78	2.30	296.59
DN80	21	10	126	5	0.0825	29.66	91.67	79.76	364.42	3737.04
Total price										**108,685**

4.2.3 *dh/L* calculations

The pressure loss per unitary length, *dh/L* is a common design parameter. If the pressure loss is high, then the investment in the pipe is well utilized, but the operating cost is high. On the other hand, if the pressure loss is low, the investment is badly utilized, but the pumping cost is low. The heat loss in a district heating pipe is higher for badly utilized pipes. The pressure loss per unit length is thus a good indicator of optimality, but not a real cost function.

4.2.4 Nodal pressure

The pressure at the nodes is determined by the element pressure loss. If a target pressure loss per unit length is defined, a target nodal pressure is also defined. The voltage law of Kirchhoff places restrictions on this, because the pressure loss along any closed path has to sum up to zero, and makes it therefore impossible to obtain target pressure loss in all the elements.

If the nodal pressure is considered an independent variable, the pressure loss per unit length can be calculated for all elements.

In this paper, the focus is on a network with a total pipe length of 4.6 km serving 10 buildings. So-called *h/L* diagrams are presented here to show the network performance. On these diagrams, the nodal head is plotted as a function of the distance from the inlet point. The *h/L* diagram for the existing network is shown on Figure 19. A similar graph for the nodal temperature is shown in

Figure 20. The network shown is the supply network with a total pipe length of 4.6 km. The return network has a similar topology, but the opposite flow direction, and is not treated in this paper. These figures show that both the head loss and the temperature drop are acceptable for the selected pipes in the network.

Figures 19 and 20 show the pipe pressure and temperature drop in the district heating network. From the pump station to the furthest lying consumer, the maximum pressure drop and temperature drop are 0.73 bar/km and 0.3°C/km, respectively, which shows the simulation results are satisfying.

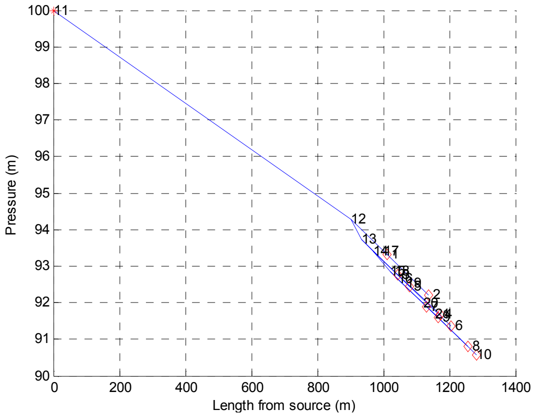

FIGURE 19: Pressure drop in a network

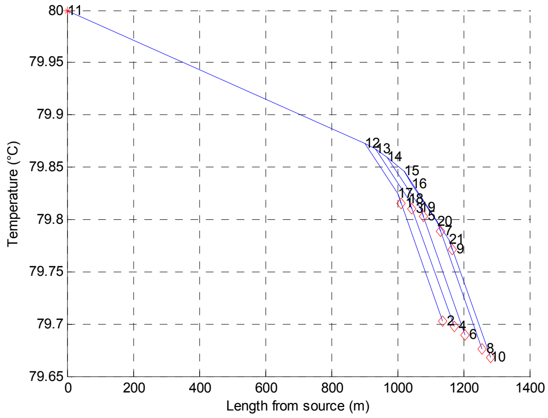

FIGURE 20: Temperature drop in a network

5. CONCLUSIONS

This paper describes geothermal district heating in Tianjin. Calculations based on two kinds of methods for a sample building were done, two kinds of simulation models were used and the results were analysed. In addition, a district heating network was set up, optimised, and the economical benefits were analysed. According to the above, the following conclusions could be obtained:

1) Heat load of buildings calculated according to Icelandic heat loss calculations and the Chinese building code, give similar results.

2) A steady-state model was used to obtain the district heating system characteristic curves.

3) A dynamic model was used to study the system when maximum water flow is limited. For the sample building in this report, 2.5 kg/s maximum mass flow is too small to maintain the indoor temperature at around 20°C despite increasing the radiator size to double or triple, so it is not an advisable way. However, 5 kg/s maximum flow with a double radiator size is preferable for the district heating system.

4) Radiator size is an important parameter that affects the indoor temperature significantly. Increasing the radiator size 2 or 3 times can improve the indoor temperature remarkably if maximum mass flow can not be changed.

5) A dynamic simulation model is a powerful tool to study the system performance, as the heating system can be controlled freely by controlling the maximum mass flow and radiator size.

6) A district heating network was set up, and pressure drop, and temperature drop of pipeline with pipe prices calculated. The maximum pressure drop and temperature drop were 0.73 bar/km and 0.3°C/km, respectively, which satisfy the simulation results.

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NOMENCLATURE

A = Heat transfer area (m^2);
C = Heat capacity of building (kJ/°C);
U = Heat transfer coefficient (W/m^2 °C);
C_p = Water heat capacity (kJ/kg °C);
g = Acceleration due to gravity (m/s^2);
V = Volume flow of infiltrate air (m^3/h);
Q = Heat load (W);
Q_o = Heat load at reference conditions (W);
Q_{supp} = Heat supply (W);
Q_{loss} = Heat loss (W);
T_w = Wall surface temperature (°C);
T_i = Indoor temperature (°C);
T_o = Outdoor design temperature (°C);
T_1 = Pipe inlet temperature (°C);
T_2 = Return temperature at pumping station (°C);
T_{i_set} = Desired temperature in dynamic model (°C);
T_{io} = Reference indoor temperature (°C);
T_{oo} = Reference outdoor temperature (°C);
T_{so} = Reference supply temperature (°C);
T_{ro} = Reference return temperature (°C);
T_s = Water supply temperature (°C);
T_r = Water return temperature (°C);
T_{c1} = Cold fluid inlet temperature (°C);
T_{c2} = Cold fluid outlet temperature (°C);
T_g = Ground temperature (°C);
T_{h1} = Hot fluid inlet temperature (°C);
T_{h2} = Hot fluid outlet temperature (°C);
ΔT_m = Logarithmic mean temperature difference (°C);
ΔT_{mo} = Logarithmic mean temperature difference at reference conditions (°C);
U_{eq} = Heat exchanger transfer matrix (W/°C)
k_l = Building heat loss factor (kW/°C);
kp = P-control parameter (kg/s °C);
m = Water mass flow (kg/s);
m_o = Reference mass flow (kg/s);
m_{ave} = Average mass flow (kg/s);
τ = Pipe transmission effectiveness;
τ_o = Pipe transmission effectiveness at reference conditions;

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APPENDIX I: Heat transfer in buildings

Heat transfer is a transient flow of thermal energy from one system to another due to temperature difference between two systems. There are three kinds of heat transfer: conduction, convection and radiation. In most cases, heat transfer is dominated by conduction and convection.

1. Conduction heat transfer

When a temperature gradient exists in a body, there is energy transfer from the high-temperature region to the low-temperature region. This energy is transferred by conduction and the heat transfer rate per unit area is proportional to the normal temperature gradient, or:

$$\frac{q_c}{A} \sim \frac{\Delta T}{\Delta x} \tag{1}$$

When a proportionality constant is inserted, this becomes:

$$q_c = -kA\frac{dT}{dx} \tag{2}$$

Equation 2 can be simplified by assuming heat transfer to be homogeneous, so the following equation is concluded:

$$\frac{q_c}{A} = \frac{k}{\Delta x}(T_1 - T_2) \tag{3}$$

where Δx = the thickness of each component of surface (m).

Rearranging Equation 3, the following relation is obtained.

$$\frac{q_c}{A} = \frac{T_1 - T_2}{\Delta x / k} = \frac{T_1 - T_2}{R_c} \tag{4}$$

where R_c is the thermal resistance due to conduction (m^2 °C/W), and is defined as follows:

$$R_c = \Delta x / k \tag{5}$$

According to Figure 5, the following relationship can be obtained:

$$q_c = \frac{T_1 - T_2}{\Delta x_1 / k_1 A} = \frac{T_2 - T_3}{\Delta x_2 / k_2 A} = \frac{T_3 - T_4}{\Delta x_3 / k_3 A} \tag{6}$$

By solving Equation 6, it can be concluded that:

$$\frac{q_c}{A} = \frac{T_1 - T_4}{\Delta x_1 / k_1 + \Delta x_2 / k_2 + \Delta x_3 / k_3} \tag{7}$$

or

$$\frac{q_c}{A} = \frac{T_1 - T_4}{R_1 + R_2 + R_3} \tag{8}$$

and

$$\frac{q_c}{A} = \frac{T_1 - T_4}{R_o + R_1 + R_2 + R_3 + R_i} \tag{9}$$

where R_o and R_i are the outside and inside film thermal resistances of the fluid, respectively.

The values for R_o and R_i are given in Tables 1 and 2 both for construction materials and metal surfaces.

TABLE 1: Inside film resistance, R_i, for construction materials

Element	**Heat direction**	**R_i (m^2.℃/W)**
Walls	Horizontal	0.12
Ceilings and floors	Upward	0.10
	Downward	0.15

TABLE 2: Outside film resistance, R_o, for construction materials for different wind speeds

Wind speed (m/s)	**≤0.5**	**0.5-5.0**	**≥5.0**
	R_o (m^2.℃/W)		
Walls	0.08	0.06	0.03
Ceilings and floors	0.07	0.04	0.02
Exposed floors	0.09	-	-

So the total thermal resistance of heat transfer is given as follows:

$$R = R_o + \sum_{k=1}^{n} R_k + R_i \tag{10}$$

Here n is the total number of layers in a multi-layer wall.

The overall heat transfer coefficient of the wall is defined as:

$$U = \frac{1}{R} \tag{11}$$

Therefore, the heat transfer from the building to the environment can be calculated according to the following:

$$q_c = UA(T_i - T_o) \tag{12}$$

where T_i and T_o are the inside and outside design temperature, respectively.

2. Convection heat transfer

To express the overall effect of convection, Newton's law of cooling is used:

$$q = hA(T_w - T_\infty) \tag{13}$$

Here the heat transfer rate is related to the overall temperature difference between the wall and fluid and the surface area *A*. The quantity *h* is called the convection heat transfer coefficient, and Equation 13 is the defining equation. An analytical calculation of *h* may be made for some systems. For complex situations it must be determined experimentally. The heat transfer coefficient is sometimes called the film conductance because of its relation to the conduction process in the thin stationary layer of fluid at the wall surface.

If a heated plate were exposed to ambient room air without an external source of motion, a movement of the air would be experienced as a result of the density gradients near the plate. This is called natural or free convection as opposed to forced convection, which is experienced in the case of the fan blowing air over a plate. In this case:

$$q_h = hA(T_w - T_o) \tag{14}$$

where h = Convection heat transfer coefficient (W/m^2℃);
A = Area of heat transfer surface (m^2);
T_w = Wall surface temperature (℃);
T_o = Air temperature (℃).

Equation 14 also can be rearranged as follows:

$$q_h = \frac{T_w - T_o}{1/hA} = \frac{T_w - T_o}{R_{conv}} \tag{15}$$

where $R_{conv} = 1/hA$ is the thermal resistance due to convection heat transfer.

3. Radiation heat transfer

In contrast to the mechanisms of conduction and convection, where energy transfer through a material medium is involved, heat may also be transferred into regions where a perfect vacuum exists. The mechanism in this case is electromagnetic radiation. This discussion is limited to electromagnetic radiation which is propagated as a result of temperature difference, this is called thermal radiation. The radiation heat flow from a surface to isothermal surroundings is calculated by:

$$q_r = \sigma A_1 F_{1-2} \varepsilon (T_1^4 - T_2^4) \tag{16}$$

where q_r = Rate of heat transfer by radiation (W);
σ = Stefan-Boltzman constant (= 5.67×10^{-8} W/m^2.K^4);
ε = Common emissivity of the objects;
A_1 = Area of surface 1 and surface 2 (m^2);
T_1, T_2 = The temperature of surface 1 and the surroundings, respectively (℃).