MODELLING OF THE URRIDAVATN LOW-TEMPERATURE GEOTHERMAL SYSTEM IN EAST ICELAND

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ABSTRACT

The Urridavatn low-temperature geothermal system in East Iceland is located underneath Lake Urridavatn. It has been utilized for space heating since December 1979. A near-vertical fracture zone appears to control the flow of water in the system and a direct downflow of colder water from the lake has caused some cooling of water from production wells. A lumped parameter model and a detailed two-dimensional numerical model of the Urridavatn geothermal system have been developed. The lumped model consists of a single reservoir block which is separated from a very large, constant pressure reservoir by a leaky aquitard. The reservoir block is fed by recharge flow from depth. A new method is used, whereby the response of the lumped model to variable production can be calculated. The response of the lumped parameter model agrees well with the observed changes in temperature, chloride-content and water level. The two-dimensional numerical model is based on a conceptual model of the Urridavatn reservoir which, in turn, is based on available geological, hydrogeological, geophysical and chemical data. The physical processes considered in the model are mass transport, as well as conductive and convective heat transfer. The numerical model is calibrated by the observed changes in temperature and water level. Predictions of the future behaviour of the reservoir, through the two models, are presented for the three production wells.

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1. INTRODUCTION

In recent years, particularly during the last decade, the use of geothermal reservoir modelling has grown significantly. Modelling has turned out to be a very effective method for analyzing data from geothermal reservoirs, as well as for estimating a geothermal field's future behaviour and its production potential. Numerous quantitative models have been developed for different geothermal fields all over the world (Bodvarsson et al., 1986).

In a broad sense, geothermal reservoir models can be divided into two categories:

- 1. Simple models: These models, although simple, are in many cases adequate idealizations of real situations (Grant et al., 1982). They have the great advantage of being simple, they do not require the use of large computers and they are inexpensive to use. But simple models can neither consider spatial variation in the properties and parameters of a reservoir nor its internal structure. According to their methods of calculation, simple models can be further divided into two subcategories: (a) distributed analytical models in which, for example, the pressure response is given by an analytical function and (b) lumped parameter models which use very few blocks to represent the geothermal system.
- 2. Numerical models: These models are very general mathematical models that can be used to simulate the geothermal reservoir in as much detail as desired. If only a few grid blocks are used, one has the equivalent of a lumped parameter model, but several hundred or thousand grid blocks can be used to simulate entire geothermal systems. But detailed numerical modelling of a geothermal reservoir is time consuming, costly and requires large amounts of field data. Numerical models can be further divided into two subcategories: (a) natural-state models developed for studies of the natural (unexploited) behaviour of geothermal systems, and (b) exploitation models developed for studies of geothermal reservoirs under exploitation (Bodvarsson et al., 1986).

In both cases, the models can only be as good as the data upon which they are based. Substantial monitoring programs are, therefore, essential.

Geothermal reservoir engineering studies have been pursued for years in Iceland (Sigurdsson et al., 1985). A few numerical models have been successfully established, such as for Krafla (Bodvarsson et al., 1984) in N-Iceland, and Nesjavellir (Bodvarsson et al., 1990) in SW-Iceland. Lumped parameter models have been developed to match the pressure response of several low-temperature geothermal fields in Iceland, among them Hamar in N-Iceland, Laugarnes in SW-Iceland, Glerardalur in N-Iceland and Laugaland in S-Iceland. These models did match the pressure data very accurately and the time required for the modelling was very short. But variations in temperature and chemical content within the systems were not taken into account (Axelsson, 1989 and 1991a).

In this report, attempts at using two modelling approaches, lumped parameter modelling and numerical modelling, to model the response of the Urridavatn low-temperature geothermal reservoir in E-Iceland, will be presented. The Urridavatn low-temperature geothermal field is located in East Iceland 3 kilometres north of the village Fellabaer (Figure 1). Geological, geophysical, geochemical and hydrogeological studies of the geothermal field have been intermittently carried out since the early sixties. Most of the geothermal system is underneath Lake Urridavatn. Eight geothermal wells, three of them production wells, have been drilled in the field. Hot water from these wells has been utilized for space heating in the villages of Egilsstaðir and Fellabaer (total population around 1500) since December 1979. After a decade of pumping, the water in the three wells, U-4, U-5 and U-8, had cooled to different extents.



FIGURE 1: Location map of the Urridavatn geothermal field

So far, no geothermal reservoir models have been developed for this geothermal system. A lot of data on temperature, chemical content and water level in the production wells are available, however. During the project described in this report a one capacitor lumped parameter model was used to simulate, and predict, temperature, chloride-content and water level in the production wells. A method was used, which considers variable flowrates, that has not been used previously for geothermal reservoirs in Iceland. In addition, a numerical model was developed using the computer code PT developed by Bodvarsson (1982) to model the temperature and water level changes during exploitation.

The work described in this report was carried out by the author during the second half of the 1991 six month training course at the UNU Geothermal Training Programme in Iceland.



FIGURE 2: Geological map of the Urridavatn geothermal field

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2. THE URRIDAVATN GEOTHERMAL SYSTEM

2.1 Geological framework

The tectonics of Iceland are controlled by the island's position on the Mid-Atlantic Ridge, with extensional features predominating. Topographically, Iceland can be divided into highlands and lowlands and most of the numerous low-temperature geothermal areas are located in the lowlands. The heat source for the low-temperature systems is believed to be the heat stored in the crustal rocks of Iceland and the regional tectonics are believed to control where circulating water can transfer the heat to the surface (Bjornsson et al., 1990), i.e. the tectonics control where low-temperature systems evolve.

The Urridavatn low-temperature geothermal field, which is one of a very few low-temperature geothermal fields in E-Iceland (Figure 1), is located in a 9.5 m.y. old basalt pile at the bottom of Lake Urridavatn. The only surface manifestations in the area were gas bubbles in the lake and holes in the winter ice. The geological formations in this region can be divided into two main types, i.e. tertiary basaltic lava flows and basaltic dykes. Figure 2 shows the dykes and faults running through the area, based on geological and geophysical surveys of the region (Einarsson et al., 1983). Three dykes have been located, running N-S along lake Urridavatn. Two of the dykes controlled the upflow of hot water into the bottom of the lake before production started. Geological mapping revealed several faults and the lake appears to be partly located in a graben. The strike of the faults in this area is N-S (Figure 2), the same as the direction of the dykes.

In general, the electrical resistivity in this region is relatively high. However, during a head-on resistivity survey in 1982 a near-vertical low-resistivity structure was found in the middle of the lake. Its strike is in a SW-NE direction (Einarsson et al., 1983) and it does not follow any of the dykes or faults previously located. This is most likely a younger fracture, or a fracture zone, which is the main aquifer, or up-flow zone, in the Urridavatn geothermal system. A well drilled to intersect this structure (well 8) confirms this; it is by far the most productive well in the area.

2.2 Hydrogeological condition

The results of a hydrogeological study performed in 1987 (Axelsson, 1987) are in agreement with the results of the head-on resistivity survey, i.e. the flow in the reservoir appears to be controlled by a vertical slab-like structure, or a fracture zone. The fracture zone is linked with Lake Urridavatn by the dykes that, prior to production, carried the hot water to the surface. This link is verified by the results of a tracer experiment performed in 1983 (Benjaminsson, 1985).

This fracture zone is most likely the structure that controls the circulation of hot water in this hydrothermal system. It is probably the conduit through which the meteoric water percolates deep into the bedrock, where it is heated by the hot rock. Then the hot water flows up along this permeable fracture, driven by the hydrostatic gradient and buoyancy. According to the results of the hydrogeological survey, the geothermal reservoir can be divided into two parts. The upper part (above 500 m) has low permeability (permeability × width = 10^{-11} m³). In this part the dykes, as well as horizontal interfaces, probably control the flow of water, in addition to the fracture zone. The lower part of the reservoir (below 500 m) has a much higher permeability (permeability × width = 1.1×10^{-10} m³). In addition to the fracture zone, other structures (dykes, interfaces) probably play only a minor role in the lower part.

2.3 History of utilization

Eight boreholes have been drilled into the Urridavatn geothermal system. Table 1 gives the basic information on these wells and Figure 3 shows their location. The first well was drilled in 1963 after a 59.5°C geothermal anomaly had been measured at the bottom of Lake Urridavatn. Among these eight boreholes, wells 4, 5, 6 and 8 have been used as production wells for the Egilsstadir District Heating Service, which started operating in December 1979.

| XX7 11 | | TTO | | | | | | |
|-----------------------------|--------|--------|-------------------------|------------------|------------------|-------------------------|--------|-------------------|
| Well | U-1 | U-2 | 0-3 | U-4 | U-5 | U-6 | U-7 | U-8 |
| Drilled | 1963 | 1963 | 1975 | 1977 | 1980 | 1981 | 1983 | 1983 |
| Depth | 116 m | 192 m | 1454 m | 1600 m | 851 m | 877 m | 344m | 1066 m |
| Туре | explo. | explo. | explo. | prod./ backup | prod./ backup | prod. | explo. | prod. |
| Main aquifers | - | - | - | 200 m 300 m | 200 m | æ., | - | 700 m -900 m |
| Small aquifers | - | - | 200 m 430 m 520 m | 450 m | 600 m | 200 m 450 m 500 m | - | - |
| Used | - | - | - | 12.79 -01.84 | 12.80 -06.84 | 08.82 -12.83 | - | 12.83 -present |
| Initial temp- erature | - | - | - | 65°C | 54°C | 61.9°C | - | 77.6°C |
| Prod. poten- tial | - | - | | 13 kg/s | 14 kg/s | 5 kg/s | - | 35 kg/s |

| TABLE 1: | Information | on | wells | in | the | Urridavatn | field |
|----------|-------------|----|-------|----|-----|------------|-------|
| | | | | | | | |

In 1979, well 4 was the only production well. It was drilled from a peninsula (built into the lake) and intersected a few aquifers above 500 m. Well 4 provided about 13 kg/s of hot water with an initial temperature of 65°C. In December, 1980, well 5 was completed. Only a very shallow aquifer was intersected at 200 m depth in addition to a minor one at 600 m depth. Well 5 produced about 14 kg/s, with an initial temperature of 54°C. The water from wells 4 and 5 cooled down very rapidly for the next few years as will be discussed in a later section.

Well 6, which was drilled in 1981, was located about 50 m further into the lake than well 4. It only produced 3-5 kg/s with a temperature of 61°C, during a 17 month production period.

Well 8 was drilled after the head-on resistivity survey had been performed and was intended to intersect the low resistivity structure at about 1000 m depth. Well 8 intersected very good aquifers between 700 and 900 meters and is, by far, the most successful well drilled in this geothermal field. It was completed in December, 1983. Since then, well 8 has been the only production well in the field yielding up to 35 kg/s of 77.5°C hot water. Since 1984, wells 4 and well 5 have only been used as backup wells. A slight decrease in temperature and some chemical dilution has also been observed for well 8 during the last several years. Table 2 shows the history of production for wells 4, 5, 6 and 8.





| Years | Average production (kg/s) | Wells |
|-------|------------------------------|--------------|
| 1980 | 13.5 | U-4 |
| 1981 | 27.0 | U-4 and 5 |
| 1982 | 27.1 | U-4, 5 and 6 |
| 1983 | 28.7 | U-4, 5 and 6 |
| 1984 | 24.0 | U-8 |
| 1985 | 25.2 | U-8 |
| 1986 | 26.3 | U-8 |
| 1987 | 26.0 | U-8 |
| 1988 | 24.3 | U-8 |
| 1989 | 19.3 | U-8 |
| 1990 | 19.6 | U-8 |

TABLE 2: History of production (Axelsson and Sverrisdottir, 1991)



2.4 Cooling and chemical dilution

The Egilsstadir District Heating Service started operating in December 1979. After that the surface manifestations disappeared, and the geothermal water produced started to cool down almost immediately, particularly the water from wells 4 and 5 (Table 3) which produce from the upper part of the reservoir. After well 5 began production the temperature in well 4 was stable for two months, then it decreased quickly to 49°C at the end of 1983. During these two months, the chloride content for well 4 increased by 4 mg/kg, then it decreased quickly from 40 mg/kg to 29 mg/kg. Figures 4 to 7 show the temperature and chloride content changes along with the flowrates for wells 4 and 5. Wells 4 and 5 have not been used since January and June 1984, respectively, because of the severe cooling. The reason for this cooling is that there are no hydrogeological barriers between the cold water in the lake and the geothermal reservoir; on the contrary, there are some permeable dykes and faults that connect the two water reservoirs. Therefore, the cold water flows easily down to the uppermost aquifers, mixing with the hot water when the pressure decreases in the aquifers due to production (Benjaminsson and Gislason, 1986). Pumping from well 8 started in December 1983. As mentioned in Section 2.2, the water produced by well 8 is from the lower part of the reservoir (depth of 700-900 m). Therefore, only about 2°C cooling was observed in well 8 from December 1983 to February 1991, during a production of 15-35 kg/s. This indicates continued downflow of cold water, but considerably less than before. The data for well 8 are shown in Figures 8 and 9.

| Well | Period of production | Temperature (°C) | Cl-content (mg/kg) |
|------|-------------------------|---------------------|-----------------------|
| U-4 | 1979.12-1984.1 | 65-49.6 | 46-29 |
| U-5 | 1980.12-1984.6 | 54-48 | 42-21 |
| U-6 | 1982.8-1983.12 | 61.9 | 38-34 |
| U-8 | 1983.12-1991 | 77.6-75.8 | 50-44 |

TABLE 3: Temperature and Cl-content changes in production wells



The suggestion that the cooling of the water from wells 4, 5 and 8 results from a downflow of cold water from Lake Urridavatn is substantiated by changes in the chemical content of the water produced. This can be clearly seen from the chloride content decreases observed in wells 4, 5 and 8 (Figures 5, 7 and 9). The chloride content in the lake is around 10 mg/kg, whereas the initial chloride content in the upper and lower parts of the reservoir were 45 mg/kg and 50 mg/kg, respectively. Because chloride takes little or no part in the chemical interactions between rock and water, it can be used to estimate the proportion of geothermal water, and water originating as cold groundwater, in the water produced. The observed chloride content is described by the equation



Here, n is the proportion of cold water in the water mixture. The fraction of water originating as cold water in the water produced by wells 4, 5 and 8 was calculated and the results are shown in Figures 10 to 12.



FIGURE 10: Mixture of cold and hot water in well 4

FIGURE 11: Mixture of cold and hot water in well 5



FIGURE 12: Mixture of cold and hot water in well 8



FIGURE 13: Production and water level for well 5

2.5 Waterlevel

Lake Urridavatn acts as a constant pressure boundary for the Urridavatn geothermal system. Therefore, the waterlevel in the wells becomes stable after several days of constant production as can be seen in Figures 13 and 14. Consequently, the long-term potential of the Urridavatn system is controlled by changes in temperature, not by long-term changes in waterlevel. However, the mass output for each well is controlled by waterlevel. Because of different permeability above and below 500 m, the waterlevel dropped about 103 m in well 5 due to 15 kg/s production, but only 30 m in well 8 due to 33 kg/s production. The stable waterlevels in wells 4, 5 and 8 observed during long-term production are listed in Table 4. Assuming that the waterlevel in each well can be described by the equation

$$h = h_{o} + bQ + cQ^{2} \tag{2}$$

where h is the waterlevel and Q the flowrate, the stable waterlevel is given by

$$h = -9 + 5.45Q + 0.3Q^2 \tag{3}$$

$$h = 4.314Q + 0.217Q^2 \tag{4}$$

$$h = -9 + 0.4755Q + 0.0203Q^2 \tag{5}$$

for wells 4, 5 and 8, respectively. The last terms in the equations describe the waterlevel drop due to turbulence in the wells, whereas the second term describes the waterlevel drop in the reservoir next to the wells.



FIGURE 14: Production and water level for well 8

| Well | Flowrate (kg/s) | Waterlevel (m) |
|------|-----------------------------------|-------------------------------|
| U-4 | 0 12.5 | -9 105-107 |
| U-5 | 0 12 14 15.5 | 0 83 103 119 |
| U-8 | 0 12.5 15 18 31 33 | -9 0 3 6 25 29 |

TABLE 4: Waterlevel and flowrate in wells 4, 5 and 8

3. SIMPLE LUMPED PARAMETER MODEL

3.1 Outline of conceptual model

The main structure determining the nature and response of the Urridavatn geothermal system is a near-vertical fracture zone (low-resistivity zone), as mentioned earlier. In addition, the geothermal system can be divided into an upper and a lower reservoir, above and below 500 m, respectively. The fracture zone connects the upper and lower reservoirs, and the upper reservoir is connected to Lake Urridavatn by a few fractures and dykes. In the natural state, hot water did flow from the upper reservoir into Lake Urridavatn, but during production cold water flows from the lake down into the upper reservoir. In addition, water from the upper reservoir flows through the fracture zone down into the lower reservoir.

The geothermal water in the upper reservoir is characterized by an initial chloride content of 40-45 mg/kg and an initial temperature of 55-65°C. The productivity of wells in the upper reservoir varies between 5 and 15 kg/s with more than a 100 m drawdown, because of low permeability. The lower reservoir is, however, highly productive. Well 8 produces up to 35 kg/s of 77°C hot water with about 20 m drawdown. Its chloride content has varied between 40 and 55 mg/kg during exploitation.

3.2 Lumped parameter model

A simple lumped parameter model was used to simulate the changes in pressure, temperature and chloride concentration observed during exploitation of the Urridavatn field. A simplified sketch of the lumped model is presented in Figure 15. The model consists of a single reservoir block which is separated from a very large, constant pressure reservoir by a leaky aquitard. The reservoir



FIGURE 15: Simplified sketch of the lumped model

block is fed by recharge flow from depth (R) and a well has been drilled into the reservoir. This well is producing at a rate of Q(t), which results in leakage through the aquitard at a rate of q(t)-R. In the natural state this leakage equals zero. Below, the governing equations for the reservoir block simulating the geothermal reservoir will be presented, as well as the equations describing the waterlevel, temperature and chloride content changes in the reservoir block derived (Kjaran, 1991; Axelsson, 1991b).

3.2.1 Fluid flow and pressure changes

The conservation of mass for this lumped model can be expressed as:

$$V\rho_{w}S\frac{dp}{dt} = \sigma(p_{o}-p) - Q + R$$
(6)

where V is the volume of the reservoir block, ρ_w the density of the geothermal water, S the reservoir compressibility, p the pressure in the geothermal reservoir, p_0 the pressure in the constant pressure reservoir and

$$\sigma = \frac{kA}{mv}$$

with k the intrinsic permeability, A the area of the geothermal reservoir, m the thickness of the aquitard and v the kinematic viscosity of the geothermal water. If one defines $p_0 = 0$, then p gives the pressure change in the reservoir. Also define

$$\lambda = \frac{\sigma}{V \rho_w S}$$

Thus:

$$\frac{dp}{dt} + \lambda p = \frac{Q - R}{V \rho_w S} \tag{7}$$

If we consider a constant flowrate, Q = constant and R = constant, then we can use the Laplace transform method to solve Equation 7. The solution, i.e. the pressure change in the reservoir, is given by:

$$p = \frac{R-Q}{\sigma}(1-e^{-\lambda t}) \tag{8}$$

In terms of waterlevel drawdown

$$\Delta h = h_o - h = \frac{Q - R}{\sigma \rho_w g} (1 - e^{-\lambda t})$$
⁽⁹⁾

The stationary waterlevel drawdown is given by:

$$\Delta h_{stat} = \frac{Q - R}{\sigma \rho_w g} \tag{10}$$

A convolution approach can be used to calculate the waterlevel drawdown for variable production rate (Q). Consider a flowrate variable in steps, i.e. $Q(t)=Q_i$ for $t_{i-1} \le t < t_i$, where i = 1, 2, ..., N. Then

$$\Delta h(t) = \sum_{i=1}^{k} (Q_i - Q_{i-1}) u(t - t_{i-1}) \qquad \text{for} \quad t_{k-1} < t < t_k \tag{11}$$

where u(t) is the unit response function given by

$$u(t) = \frac{1}{\sigma \rho_w g} (1 - e^{-\lambda t}) \tag{12}$$

Defining $h_k = h(t_k)$

$$h_{k} = h_{o} - \frac{1}{\sigma \rho_{w} g} \sum_{i=1}^{k} (Q_{i} - Q_{i-1})(1 - e^{-\lambda(t_{k} - t_{i-1})}) + CQ_{k}^{2} \qquad \text{for } k = 1, 2, ..., N$$
(13)

Here CQ^2 is a term describing the turbulence waterlevel drop in production wells. Note that $t_0=0$ and $Q_0=R$. A computer program was written to calculate the pressure response of the lumped model during production according to Equation 13.

3.2.2 Chemical changes

The conservation of a chemical substance that does not react with the reservoir rock, for example chloride, is given by

$$\rho_{w}V\phi\frac{dc}{dt} = qc' + Rc_{R} - Qc \qquad (14)$$

where ϕ is the porosity of the geothermal reservoir, c is the chemical concentration in the geothermal reservoir, c' is the chemical concentration in the constant pressure reservoir, c_R is the chemical concentration of the recharge from depth and q, the leakage through the aquitard, is given by

$$q = (Q - R)(1 - e^{-\lambda t}) \tag{15}$$

Because

$$\lambda >> \frac{Q}{Ab\phi\rho_w}$$

one can approximate:

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q = Q - R

If, in addition, one assumes that $c_R = c_0$, the initial concentration in the reservoir, then Equation 14 becomes

$$\frac{dc}{dt} + \alpha Qc = \alpha \left(Q - R\right)c' + \alpha Rc_o \tag{16}$$

where

$$\alpha = \frac{1}{\rho_w V \phi}$$

The Laplace transform method is employed for a constant production Q, resulting in

$$c = \left(\frac{Q-R}{Q}c' + \frac{R}{Q}c_o\right)\left(1 - e^{-\alpha Qt}\right) + c_o e^{-\alpha Qt}$$
(17)

For a variable flowrate (defined in Section 3.2.1):

$$c_{i} = c(t_{i}) = \left(\frac{Q_{i} - R}{Q_{i}}c^{\prime} + \frac{R}{Q_{i}}c_{o}\right)\left(1 - e^{-\alpha Q_{i}(t_{i} - t_{i-1})}\right) + c_{i-1}e^{-\alpha Q_{i}(t_{i} - t_{i-1})}$$
(18)

for i = 1, 2, ..., N

A computer program was written to calculate the chloride content response of the lumped model during production according to Equation 18.

3.2.3 Energy balance and temperature

The conservation of energy, or heat, in the lumped model can be expressed by:

$$V(\rho r)\frac{dT}{dt} = r_w(qT' - QT + RT_r)$$
(19)

where (ρr) is the volumetric heat capacity of the reservoir, r_w the heat capacity of water, T the temperature in the geothermal reservoir, T' the temperature in the constant pressure reservoir and T_R the temperature of the recharge from depth. Making the same approximations as in Section 3.2.2

$$\frac{dT}{dt} + \beta QT = \beta (Q - R) T' + \beta T_o R$$
⁽²⁰⁾

with

$$\beta = \frac{r_w}{V(\rho r)}$$

21

Here, pr, the density times heat capacity of the geothermal reservoir, is given by

$$\rho r = \rho_w r_w \phi + \rho_r r_r (1 - \phi)$$

where r_w and r_r are heat capacity of the geothermal water and the rock, respectively. As before, Equation 20 can be solved by the Laplace transform method. A constant flowrate solution is given by:

$$T = \left(\frac{Q-R}{Q}T + \frac{R}{Q}T_o\right)\left(1 - e^{-\beta Qt}\right) + T_o e^{-\beta Qt}$$
(21)

But for a variable flowrate:

$$T_{i} = T(t_{i}) = \left(\frac{Q_{i} - R}{Q_{i}}T + \frac{R}{Q_{i}}T_{o}\right)\left(1 - e^{-\beta Q_{i}(t_{i} - t_{i-1})}\right)$$
$$+ T_{i-1}e^{-\beta Q_{i}(t_{i} - t_{i-1})} \qquad for \quad i = 1, 2, ..., N$$
(22)

A computer program was written to calculate the temperature response of the lumped model during production according to Equation 22.

3.3 Results of lumped parameter modelling

This lumped parameter model is the first quantitative model developed for the Urridavatn geothermal system. Lots of data are available for calibrating the model, in particular data on changes in temperature and chemical concentration. Long-term pressure response data is, however, limited. Pressure response data from a hydrogeological test (Axelsson, 1987) carried out in August 1987 was, therefore, used to calibrate the lumped parameter model. The temperature and chloride concentration in the constant pressure reservoir, the initial values for the geothermal reservoir as well as the quantity of recharge from depth were varied until a good fit to the observed changes was obtained for the different wells (wells 4, 5 and 8). A comparison of the

TABLE 5: Parameters of lumped parameter models, part 1

| Well | Inflow from depth (kg/s) | Initial temperature (°C) | Temperature in constant pressure reservoir (°C) | Initial Cl- content (mg/kg) | Cl-content in constant pressure reservoir (mg/kg) |
|------|-----------------------------------|--------------------------------|---|--------------------------------------|---|
| U-4 | 7 | 65 | 41 | 44 | 22 |
| U-5 | 7 | 54 | 15 | 34 | 18 |
| U-8 | 15 | 77 | 53 | 52 | 35 |

observed and calculated data is presented in Figures 16 to 25 and the parameters of the lumped parameter model are presented in Tables 5 and 6. The results in Figures 16-25 show that this



FIGURE 18: Observed and calculated temperature for well 5 (lumped model)

FIGURE 19: Observed and calculated Clcontent for well 5 (lumped model)

model can quite successfully match the observed history of the Urridavatn system. The parameters in Table 6 also provide some information on the physical properties and size of the geothermal system.

Generally speaking, production temperature and chemical concentration cannot be simulated very well by lumped models. This fundamental flaw of lumped parameter models results from their failure to describe spatial variations. For the Urridavatn geothermal system, however, the calculated values fit the observed chloride content and production temperature quite well. But the fact that the lumped parameter model cannot describe spatial variation is reflected in Figure 16. For well 4, the calculated temperature does not fit the observed temperature during December 1980 and January 1981, the first two months after production started from well 5. This probably reflects the fact that the feedzones in the two wells are not at the same depth.





FIGURE 22: Observed and calculated waterlevel for well 5 (lumped model)



It should be pointed out that, for well 5, the constant pressure reservoir in the model simulates Lake Urridavatn and the groundwater system immediately below the lake. This is reflected in a low temperature and chloride content in this part of the model for well 5 (Table 5). For well 4, the constant pressure reservoir in the model probably simulates the lake, the groundwater system as well as the uppermost part of the geothermal system (above 200 m approximately). For well 8, the constant pressure reservoir simulates all of the upper reservoir (above 500 m) as well as the lake and groundwater system. For wells 4 and 5, the reservoir block in the model simulates the upper part of the geothermal reservoir whereas, for well 8, the reservoir block simulates the deeper part of the actual reservoir (below 500 m).



TABLE 6: Parameters of lumped parameter models, part 2

| Well | Volume of research | rvoir (10 ⁶ m ³) | Permeability of | Porosity |
|------|--|---|---|----------|
| | Temperature and chloride content model | Pressure model | aquitard (10 ⁻¹³ m ²) | |
| U-4 | 1.67 | - | - | 0.676 |
| U-5 | 22.4 | 2.04 | 1.55 | 0.033 |
| U-8 | 30.5 | 97.3 | 5.87 | 0.017 |

TABLE 7: Predicted waterlevel in production wells

| Production | | Waterlevel (m) | |
|------------|-------|----------------|------|
| (kg/s) | U-4 | U-5 | U-8 |
| 10 | 75.5 | 64.84 | -2.2 |
| 15 | 140.2 | 113.5 | 2.7 |
| 20 | 220.0 | 173.1 | 8.6 |
| 30 | | - | 23.5 |
| 40 | - | - | 42.5 |
| 50 | - | - | 65.5 |
| 60 | - | - | 92.6 |

The lumped models for wells 4, 5 and 8 were used to predict the reservoir behaviour in the future (until the year 2000). The results are presented in Figures 26 to 29. The predicted waterlevel changes are presented in Table 7. If the average production from well 8 equals 30 kg/s for the next 10 years, the temperature will not reach a steady-state but will decrease to 71°C. The average



FIGURE 26: Past and predicted future temperature for well 4 according to the lumped model



FIGURE 27: Past and predicted future temperature for well 5 according to the lumped model

waterlevel will be at 23.5 m and the chloride content around 43.5 mg/kg. This suggests that the inflow from upper layers will reach about 50%, based on the chloride concentration of the constant pressure reservoir, 35 mg/kg. Production from the upper reservoir (wells 4 and 5) for long periods is not advisable, since it will accelerate the cooling in the lower reservoir.



FIGURE 28: Past and predicted temperature for well 8 according to the lumped model



FIGURE 29: Past and predicted future Cl-content for well 8 according to the lumped model

4. TWO-DIMENSIONAL NUMERICAL MODEL

4.1 General overview

As mentioned earlier, simple lumped models have some serious disadvantages, for example the fact that they do not consider spatial variations in temperature, pressure and reservoir properties. The fact that three different models had to be used to simulate the response of the three different production wells (4, 5 and 8) highlighted the need to consider spatial variations in models of the Urridavatn geothermal reservoir. A two-dimensional numerical model was, therefore, developed to provide a better physical representation of the recharge of cold and hot water and to improve the reliability of predictions of the future performance of production wells in the area. A computer code, named PT, was used to solve the equations for mass and heat transfer in the model. This code was developed for liquid-phase porous reservoirs and can solve one-, two- or three-dimensional problems (Bodvarsson, 1982).

4.2 Numerical formulation

The code PT employs the Integrated Finite Difference Method for formulating the governing equations (Bodvarsson, 1982). The model is divided into arbitrarily-shaped nodes, or blocks. For an arbitrary node n, the governing equations are written as follows (see nomenclature for symbols):

Mass balance

$$V_{n}\rho_{w}[S_{n}\frac{\Delta p}{\Delta t} - \alpha_{n}\phi_{n}\frac{\Delta T}{\Delta t}] = \sum_{m} \left(\frac{kA}{\nu}\right)_{n,m}$$

$$\times \left[\frac{p_{m}-p_{n}}{D_{-n}+D_{mn}} - \eta_{g}\rho_{g}g\right] + \left(G_{f}V\right)_{n}$$
(23)

Energy balance

$$[(\rho r) V]_{n} \frac{\Delta T_{n}}{\Delta t} = \sum_{m} \left[\frac{(\lambda A)_{n,m}}{D_{n,m} + D_{m,n}} (T_{m} - T_{n}) + \left(\frac{r_{w} Ak}{v} \right)_{n,m} (T_{m,n} - T_{n}) \left(\frac{p_{m} - p_{n}}{D_{n,m} + D_{m,n}} - \eta_{g} \rho_{g} g \right) \right] + (G_{h} V)_{n}$$
(24)

These equations can be combined for simultaneous solution in a single matrix equation.

$$[A] \mathbf{X} = \mathbf{b} \tag{25}$$

The coefficients in the matrix [A] are, in general, functions of temperature and pressure and, therefore, the equations are nonlinear. The vector X contains the unknowns $[\Delta p \text{ and } \Delta T]$ and the vector **b** represents the known explicit quantities. The sets of nonlinear equations are solved basically by using LU decomposition, Gaussian elimination and an iterative scheme for the nonlinear coefficients (Bodvarsson, 1982).



FIGURE 30: Block layout for the two-dimensional model at Urridavatn

4.3 The model

Based on the conceptual model of the Urridavatn geothermal system and the results of the lumped parameter modelling, a 2000 m long and 1500 m deep cross-section along the low resistivity fracture was chosen. The grid for the model is shown in Figure 30. It consists of 8 layers (layers A-H) with 13 blocks in each layer, except for the uppermost layer (layer I) which has 5 blocks.

The initial permeability distribution of the model was based on available permeability estimates for the Urridavatn reservoir, for example results of the hydrogeological test in 1987 (Axelsson, 1987). Layers E, F, and G have a relatively high permeability to model the deep reservoir, whereas layers B and C have somewhat lower permeability (the upper reservoir). Layers A and D have a low permeability, except for several high permeability blocks (A5-A10,D5-D6), which are supposed to enable the circulation between the two reservoirs and the discharge to the surface. The model is completed by defining very large blocks in the top layer (layer I) to simulate Lake Urridavatn. A high permeability and porosity were assigned to this layer, such that temperature and pressure changes would be negligible in spite of inflow of hot water from the geothermal system.

This reservoir model has a finite volume. Therefore, boundary conditions must be specified. There are two types of boundary conditions, either the temperature and pressure are specified or heat and mass fluxes are given. Only the former type is considered in this model. Constant pressure and temperature were specified in columns 1 and 13 and in layer H where large volume blocks were used to control the recharge to the geothermal system. Three sinks, which represent the production wells 4, 5 and 8, were specified in blocks B7, C9 and G3.

4.4 Results of numerical modelling

The two-dimensional numerical model was calibrated by obtaining a good match to the observed temperature and pressure changes. The parameter adjustments involved varying the permeability and porosity of each block, as well as the heat capacity. The fluid flow in the model was adjusted by varying the permeability until the flow was in agreement with the conceptual model of the Urridavatn reservoir. The recharge of hot water was controlled by adjusting the permeability of the boundary blocks on the two sides of the model and the inflow of cold water by changing the permeability in the vertical high permeability channel. The permeability in the vertical channel is one of the most sensitive parameters of the model since it not only controls the pressure changes but also the temperature changes in the two reservoirs. This was a lengthy process of trial and error. If a good match to the production temperature and pressure was not attained, the simulated parameters were adjusted again and the whole cycle of simulation was repeated.

The grid for the numerical model is shown in Figure 30 along with the permeability of different parts of the model. The basic properties of the model are presented in Table 8 and the heat capacity, porosity and permeability of different parts of the model in Table 9. The response of the numerical model agrees well with the production history as shown in Figures 31 to 36. The need for a high permeability channel suggests that the cold water downflow into the reservoirs is not uniform and the low porosity and heat capacity of the channel suggest a downflow through a fracture-like channel. This must be the main reason for the fact that the reservoir started to cool down immediately after production started.



FIGURE 31: Observed and calculated temperature for well 4 (two-dim. numerical model)

The model predicts a slow decrease in pressure and temperature of the deeper reservoir up to the year 2000, as shown in Figures 37 and 38. The two-dimensional numerical model is very useful for gaining information about the vertical permeability distribution and the mechanism of cold water inflow into the Urridavatn geothermal system. But it could not model the production temperature for well 4, which produced hot water not only from a 500 m level aquifer considered in the model, but also from a 200 m level aquifer. Therefore, the calculated temperature for well 4 does not match very well the observed temperature after production started from well, which produced hot water from a 200 m aquifer only.

| Density of the rock (kg/m ³) | 2800 | |
|--|-----------------------|--|
| Heat capacity of water (J/kg°C) | 4200 | |
| Compressibility of the rock(Pa ⁻¹) | 2.0×10 ⁻¹¹ | |
| Thickness (m) | 50 | |
| Thermal conductivity (J/ms°C) | 2.1 | |

TABLE 8: Basic properties of the numerical model



FIGURE 32: Observed and calculated waterlevel for well 4 (two-dim. numerical model)



FIGURE 33: Observed and calculated temperature for well 5 (two-dim. numerical model)



FIGURE 34: Observed and calculated waterlevel for well 5 (two-dim. numerical model)



FIGURE 35: Observed and calculated temperature for well 8 (two-dim. numerical model)



FIGURE 36: Observed and calculated waterlevel for well 8 (two-dim. numerical model)

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| Blocks | Permeability (10 ⁻¹³ m ²) | Porosity | Heat capacity of the rock (J/kg°C) |
|---|---|----------|--|
| E3,E4,E6- E11,F3,F4,F6- F11,G3-G11 | 59.3 | 0.08 | 8000 |
| B5,B6,C5,C6,D5,D6, E5,E6,F5,F6 | 17.0 | 0.001 | 1 |
| A5-A10,B9 | 12.0 | 0.09 | 1000 |
| E1,E2,E12,E13,F1, F2,F12,F13,G1,G2, G12,G13 | 3.50 | 0.35 | 900 |
| A1-A4,A11-A13 | 1.50 | 0.05 | 800 |
| C2-C4,C7-C12 | 0.86 | 0.13 | 800 |
| B2-B4,B7,B8,B10- B12 | 0.355 | 0.10 | 4500 |
| B1,C1,B13,C13 | 0.10 | 0.25 | 900 |
| D1-D4,D7-D13,H1- H13 | 0.01 | 0.05 | 900 |
| I1 - I5 | 1000 | 0.98 | 4200 |

TABLE 9: Properties of different parts of the numerical model







FIGURE 38: Past and predicted future waterlevel for well 8 according to the two-dimensional numerical model

5. CONCLUSIONS

The main conclusions of the study are as follows:

- 1. The Urridavatn low-temperature geothermal system can be divided into an upper reservoir between 200 and 500 m depth, which has a low permeability, and a lower reservoir below 500 m, which has a much higher permeability. A near-vertical fracture zone controls the flow of water in both reservoirs. The reservoir temperatures are between 55 and 65°C in the upper reservoir and about 78°C in the deeper reservoir.
- 2. The water from the production wells in the Urridavatn-field has cooled down during the last decade. The cause for this is a direct downflow of colder water from lake Urridavatn, through fractures and dykes, into the geothermal system.
- 3. A new method of lumped parameter modelling, which considers variable production rates, was successfully used to simulate the changes in temperature and chemical content during exploitation of the Urridavatn geothermal system.
- 4. The calculated responses of the lumped model are in a good agreement with the production history of the Urridavatn field. According to the lumped model the predicted temperature for well 8 will go down to 71°C, assuming a flowrate of 30 kg/s up to the year 2000.
- 5. A two-dimensional numerical model was developed based on the current conceptual model of the field. It was calibrated by the observed changes in temperature and waterlevel.
- 6. A highly permeable channel with a very low heat capacity and porosity was required in the two dimensional numerical model to simulate the cooling of well 8. This suggests that the cold water downflow into the reservoir is not uniform but goes through a fracture of limited extent.
- 7. A detailed three-dimensional reservoir model should be developed for the Urridavatn geothermal reservoir in order to predict more accurately the future behaviour of the geothermal system.

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NOMENCLATURE

Α - surface area of geothermal reservoir (m²) A_{n,m} - area of interface between nodes n and m (m^2) C - turbulence parameter (m/(kg/s)²) c' - chemical concentration in constant pressure reservoir (Lake Urridavatn) (mg/kg) - chemical concentration of recharge from depth (mg/kg) CR c₀ - initial chemical concentration in geothermal reservoir (mg/kg) chemical concentration in geothermal reservoir (mg/kg) C c_w, c_r - compressibility of water and rock, respectively (P_a^{-1}) D_{n.m} - distance from nodal point of node n to interface (m) $D_{m,n}$ - distance from nodal point of node m to interface (m) - acceleration due to gravity (m/s²) g Gf - mass source/sink (kg/m³s) Gh - heat source/sink (J/m³s) - waterlevel in constant pressure reservoir (m) ho h - waterlevel in geothermal reservoir (m) Δh - waterlevel drawdown (m) k - intrinsic permeability (m²) - permeability at interface between nodes n and m (m²) k_{n.m} thickness of aquitard (m) m - pressure in constant pressure reservoir (Pa) Po - pressure in geothermal reservoir (Pa) p - pressure in node m (Pa) Pm - pressure in node n (Pa) Pn - pressure change in node n (Pa) Δp_n - production rate (kg/s) Q - flowrate through the aquitard (kg/s) q R - hot water inflow from depth (kg/s) rw, rr - heat capacity of geothermal water and rock, respectively (J/kg°C) S - compressibility of geothermal reservoir = $c_w \phi + c_r (1-\phi) (Pa^{-1})$ S_n - compressibility of node n (Pa⁻¹) - time (s) t T - temperature in constant pressure reservoir (°C) TR temperature of hot recharge (°C) T₀ - initial temperature in geothermal reservoir (°C) T - temperature in geothermal reservoir (°C) Tm temperature in node m (°C) T_n - temperature in node m (°C) T_{m,n} - temperature in the interface of node m and node n (°C) ΔT_n - temperature change in node n (°C) u - unit response function (m/(kg/s)) V - volume of geothermal reservoir (m³) V_n - volume of node n (m³) - kinematic viscosity of geothermal water (m²/s) υ - density of the interface between node n and node m (kg/m³) $\rho_{n,m}$ ρ_w, ρ_r - density of geothermal water and rock, respectively (kg/m³) - density × heat capacity of geothermal reservoir (J/m^{3o}C) (ρr) - fluid density at the interface (kg/m₃) ρg - expansivity of node n (°C⁻¹) α_n - porosity of geothermal reservoir Φ - direction cosine for the gravity term η_g - thermal conductivity of the interface between nodes n and m (J/ms°C) λ_{m.n}

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