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GEOTHERMAL RESERVOIR PHYSICS

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CONTENTS

INTRODUCTORY REMARKS

A considerable number of physical and mathematical problems have to be tackled in order to obtain a reasonable grasp of the processes involved in geothermal reservoir physics. True, petroleum reservoir engineering that is now a fairly well developed technology provides an extensive basis for the development of the parallel discipline of geothermal reservoir physics. However, although the problem setting may appear rather similar, one has to apply caution in the adoption of the principles and results of the petroleum sciences to the geothermal setting. The principal problems arise from the much greater involvement of geothermal resource physics with elastomechanical and thermodynamic problems than petroleum reservoir engineering. The collection of papers that follows has been assembled in order to highlight and discuss at lenght, some of the physical problems that we have to consider in the geothermal reservoir sciences. These problems are both in reservoir testing, as well in the production of geothermal fluids such as liquid water, steam, or mixtures thereof.

Eight papers appear in this report. Four of the papers have not been published before and appear here for the first time, as individual chapters. Four papers already published in journals are also included in this report for completeness, and appear as appendices.

Geothermal reservoir testing is almost invariably carried out on the basis of studies of the propagation of controlled or natural pressure signals through the reservoir formations. "A few remarks on liquid reservoir testing" and "Confined fluids as strain meters" give an overview of the theory of the propagation of test signals through ordinary porous reservoir formations without considering elastomechanical interactions between the fluid and the formation. The basic field equations of propagation are set forth and the basic input and derived parameters are defined.

Since we are here concerned with rather unconventional problems, we have had to introduce some unconventional terminology. The case in point is the concept of hydroelastic phenomena. These involve the elastomechanical interaction between the formation and the mass of the reservoir liquid. The elasticity of the formation provide the restoring force field and the liquid mass is the oscillating mass.

It is now well known that most geothermal reservoirs are embedded in fractured formations such that the porous reservoir models may be rather unreliable and lead one astray. The propagation of pressure signals through fractures with elastic walls is then an important topic to be considered. The thrust of the paper entitled "Hydroelastic pressure field propagation in fracture spaces" concerns this problem setting. Again, the basic equations of propagation and the parameters involved are set forth and discussed.

The papers in the collection entitled "Mechanics of two types of hydroelastic formation fields" and "Hydroelastic oscillations in borehole cavity systems" further elaborate on the hydroelastic phenomena of interest in the present context. The latter paper describes hydroelastic oscillations in their purest form. Here we assume that a borehole filled with a liquid mass can oscillate when it is connected to an elastic cavity. The mass of the liquid column oscillates with the help of the elastic restoring force of the cavity. The paper elaborates on the various field conditions where such phenomena can be observed.

The paper entitled "Dynamics of borehole-fracture systems and the detection of fractures by acoustic techniques" considers a rather useful aspect of hydroelastic phenomena, that is, how they can be applied to detect the position of fractures and even furnish information on their width. This is of some interest in practical geothermal reservoir work.

In the paper "Linearization techniques and surface operators in the theory of unconfined aquifers," we consider the particular mathematical problems that are encountered in work on liquid phase flows in porous media aquifers with a free liquid surface. This type of problem setting does, in particular, lead to the introduction of Laplacian operators of a fractional order which is highly unusual in mathematical physics.

The paper "The exergy of thermal water" turns to a topic of thermodynamics in geothermal engineering. The exergy is a very useful concept that has long been neglected. It gives the maximal amount of mechanical energy that can, at ideal conditions, be extracted from a unit mass of thermal water at a given temperature. Just as the enthalpy gives the amount of heat energy that can be extracted per unit mass, the exergy is a function of state. Because of the second law of thermodynamics, the exergy is considerably smaller than the enthalpy, when both properties are taken at the same temperature.

I hope that this collection of papers will be of interest to students of geothermal reservoir physics.

I am thankful to Dr. Jón-Steinar Gudmundsson, the Director of the Geothermal Training Programme of the United Nations University at the National Energy Authority in Iceland for promoting the publication of this report.

Gunnar Bödvarsson

A FEW REMARKS ON LIQUID RESERVOIR TESTING

Abstract

The mechanics of liquid reservoir testing for principal geometric and physical parameters is based on interstitial fluid pressure fields that are governed by partial differential equations of the diffusion or parabolic type. Various types of standard and non-standard boundary conditions are to be adjoined to the equations. The free-liquid surface condition is one of the most interesting non-standard conditions entering into consideration. The basic equations can be solved by a number of mathematical procedures that yield different types of solutions. The iteration type Taylor-series approach is suitable for the computation of short-term field development. The eigenfunction approach is more adopted to obtaining long-term types of solutions. Various types of diffusion field characteristic parameters such as skin-depth, penetration depth and relaxation time are of considerable practical relevance. Particular methods are required to treat problems that arise due to fracture-dominated situations. Analog to other types of geophysical field exploration problems, the interpretation of reservoir test data encounters severe problems due to non-uniqueness. The most extreme example is the considerable insensitivity of pressure-field testing to the dimensionality of the interpretation model.

Introduction

The primary targets of water, petroleum and geothermal resource exploration are total volume, flow and capacitive characteristics, chemical quality and in the particular case of geothermal resources, the reservoir temperature field. While geophysical exploration by surface methods may furnish some data on reservoir structure, temperature fields and give indications as to the reservoir volume, they furnish practically no information on the fluid conductivity and other production parameters. Such information will generally have to be obtained by tests performed within the reservoir, primarily by production and/or interference tests on sufficiently deep wells. Reservoir testing is therefore one of the most important tasks in an exploration program.

In principle, reservoir testing has much in common with conventional geophysical exploration. Although the physical fields applied are to some extent different, we face the same type of selection between controlled and natural drives, forward and inverse problem setting, etc. The basic philosphy (Bodvarsson, 1966) is quite similar.

Below, we will discuss some aspects of reservoir testing where the fluid conductivity is the primary target. Although well known material, it is in this context useful to commence by reviewing a few fundamentals of the theory of Darcy type flow in slightly compressible porous formations. This material to be presented below, will help to put other results in a proper perspective.

Basic relations governing the pressure field in slightly compressible Darcy type formations

Diffusion equation. Let $p(t,P)$ be the pressure field at time t and at the point P in a Darcy type domain B with the stationary boundary surface Σ. Consider a general setting where the permeability κ is a spatially variable linear matrix operator but the kinematic viscosity of the fluid ν is taken to be constant. It is convenient to introduce the fluid conductivity operator $c = \kappa/\nu$ and express Darcy's law

$$\vec{q} = -c\nabla p \tag{1}$$

where $\vec{q}$ is the mass flow density. Moreover, let ρ be the fluid density, s the capacitivity or storage coefficient of the formation and f be a source density. Combining (1) with the equation for the conservation of mass, we obtain the diffusion equation for the pressure field (Bodvarsson, 1970)

$$\rho s \partial_t p + \Pi(c)p = f \tag{2}$$

where $\Pi(c) = -\nabla(c\nabla)$ is a generalized Laplacian operator. Appropriate boundary conditions that may be of the Dirichlet, Neumann, mixed or more complex convolution type, have to be adjoined to equation (2) The case of a homogeneous/isotropic/isothermal formation results in the simplication $\Pi(c) = c\Pi = -c\nabla^2$ where c is a constant. Moreover, stationary pressure fields satisfy the potential equation

$$\Pi(c)p = f. \tag{3}$$

Eigenfunctions of the Laplacian. The eigenfunctions $u_n(P)$ of $\pi(c)$ in B with (2) satisfy the equations

$$\Pi(c)u_n = \lambda_n u_n, \quad n = 1,2, \ . \ . \ . \tag{4}$$

where the constants λ are the eignvalues and the boundary conditions on Σ are homogeneous of the same type as those satisfied by p(t,P) in (2) and (3).

Types of solutions. It is of interest to consider some general expressions for the solutions of equation (2) above. The key to the equation is the causal impulse response or Green's function G(P,Q,t) which represents the pressure response of the causal system to an instantaneous injection of a unit mass of fluid at t = 0+ at the source point Q. This function satisfies the same boundary conditions as the eigenfunctions $u_n(P)$. Solutions to (2) in the case of a general source density f(t,P), non-causal initial values and general boundary conditions can then be expressed in terms of integrals over the Green's function (Duff and Naylor, 1966).

Two fundamental types of expressions for the Green's function are available. First, in the case of simple layered domains B with a boundary Σ composed of a few plane faces, G(P,Q,t) can be expressed as a sum (or integral) over the fundamental whole space source function,

$$G_o(P,Q,t) = (8\rho s)^{-1}(\pi at)^{-3/2}\exp(-r_{PQ}^2/4at)U_+(t) \tag{5}$$

and their images. The symbol $U_+(t)$ is the causal unit step function, $a = c/\rho s$ the diffusivity, and r_{PQ} is the distance from Q to P. Whenever applicable, sums of this type represent the most elementary local and/or global expressions for G(P,Q,t).

Second, the Green's function can be expanded in a series (or integral) over the eigenfunctions of $\Pi(c)$. If ρ and s are constants, then

$$G(P,Q,t) = (1/\rho s) \sum_n u_n(P) u_n^*(Q) \exp(-\lambda_n t/\rho s). \qquad (6)$$

The series expansion (6) is of a more general applicability than solutions of the type based on the fundamental source function (5). However, because of quite poor convergence properties, (6) is largely of a more global long-term relevance. It is less suited for the computation of local values. The formal link between the two types of solution (5) and (6) is provided by the Poisson summation formula (Stakgold, 1967).

A different type of solution of (2) that is of interest in the present context can be obtained by operational methods. Limiting ourselves to the pure initial value problem with $p(0,P) = p_0(P)$ in the case of an infinite domain, we can, since ρ, s and $\Pi(c)$ are independent of t, formally express the solution of the homogeneous form of (2) as

$$p = \exp[-t\Pi(c)/\rho s] p_0 \qquad (7)$$

where the exponential operator is to be interpreted as a Taylor series in the operator $\Pi(c)$

$$\exp[-t\Pi(c)/\rho s] = 1 - [t\Pi(c)/\rho s] + (1/2)[t\Pi(c)/\rho s]^2 \ . \ . \ . \qquad (8)$$

The series represents an iteration process where the convergence is limited to (properly defined) small values of t. The practical applicability is therefore fundamentally different from (6). Moreover, it is of considerable interest that rather general situations with regard to $\Pi(c)$ can be admitted in (7) and (8).

A number of other analytical and/or numerical techniques are available for solving (2). These include the path-integral technique of the Feynman-Kac type (Simon, 1979), compartmentalization or lumping and, as a matter of course, a series of numerical techniques.

Nonstationary boundaries: effects of a free liquid surface

The presense of a free liquid surface in a reservoir requires the introduction of a rather complex non-stationary surface boundary condition. Let Σ now represent the free liquid surface at equilibrium and Ω be the free surface in a perturbed state. The boundary Ω is a surface of constant pressure which without loss of generality can be taken to vanish. The free surface condition (Lamb, 1932) is then expressed

$$Dp/Dt|_{p=o} = 0 \tag{9}$$

where D/Dt is the material derivative. This is an essentially non-linear condition which leads to a much more complex problem setting. Losing the principle of superposition the construction of solutions to the forward problem becomes a difficult task.

Bodvarsson (1977) has shown that when Ω deviates only little from Σ, (9) can be simplified and linearized. For this purpose, we place a rectangular coordinate system with the z-axis vertically down such that the (x,y) plane coincides with Σ. Moreover, let the amplitude of Ω relative to Σ be u and the scale of the undulation of Ω be L. Then provided $|u/L| \ll 1$, the condition (9) can be replaced by the approximation

$$(1/w)\partial_t p - \partial_z p = 0, \tag{10}$$

where $w = cg/\phi$ is a new parameter, namely, the free sinking velocity of the pore liquid under gravity (g = acceleration of gravity). Under these circumstances, the solution of the forward problem is obtained by constructing a solution to (2) which satisfies (10) at the free surface and appropriate conditions at other sections of the reservoir boundary.

The presence of a first order derivative with respect to time in the free-surface condition (10) obviously leads to an additional relaxation process analog to the purely diffusive phenomena associated with the first order time derivative in the basic equation (2). As we shall conclude below, the individual time scales of the two phenomena are, however, different.

For the sake of brevity, we shall limit the present discussion to the simplest but practically quite relevant case of the semi-infinite liquid saturated homogeneous, isotropic and isothermal half-space. To consider the pure free-surface related phenomena, we eliminate pressure field diffusion by neglecting the compressibility of the liquid/rock system. In this setting we can combine the potential equation (3) and the surface condition (10) in one single equation confined to the Σ plane (Bodvarsson, 1984) which expressed in terms of the fluid surface amplitude $u(t,x,y) = p/\rho g$ takes the form

$$(1/w)\partial_t u + \Pi_2^{\frac{1}{2}} u = f/\rho gc \tag{11}$$

where $\Pi_2^{\frac{1}{2}} = (-\partial_{xx}-\partial_{yy})^{\frac{1}{2}}$ is the square root of the two-dimensional Laplacian and f is an appropriately defined source density. To obtain the pressure field in the space z>0, the boundary values derived from (11) have to be continued into the lower half-space on the basis of standard potential theoretical methods. The fractional order of the Laplacian in (11) is quite unusual, but the operator is well defined and poses no mathematical problems.

Some solutions of equations (11) of practical interest have been obtained by Bodvarsson (1977). Confining ourselves again to the simple semi-infinite half-space, the most important result is given by the causal impulse-response functions G(S,Q,t) which represents the response of the surface amplitude at the point S = (x,y) in Σ and time $t \geq 0+$ to an instantaneous injection of a unit mass fluid at a point Q in the half-space at time t=0+. The system is assumed to be in equilibrium for $t \leq 0$. Let Q = (0,0,d), the resulting expression for the surface amplitude is

$$G(S,Q,t) = (1/2\pi\phi\rho)(wt+d)U_+(t)/[x^2+y^2+(wt+d)^2]^{3/2} \tag{12}$$

where $U_+(t)$ is the causal unit-step function. The impluse response is essentially the key to the solution of (11) for more general conditions. The pressure field in the half-space is obtained from (12) by a simple continuation technique where the singularity at Q has to be taken into consideration. The long term response of the surface amplitude to a periodic source function at Q is of particular interest in the present context. Let the mass flow injected at Q = (0,0,d) take the form exp(-iωt). The amplitude of the frequency response is then obtained by

$$F(S,Q,\omega) = \int_0^\infty G(S,Q,\tau)\exp(i\tau\omega)d\tau \tag{13}$$

The present results on the dynamics of the free surface amplitude provide the basis for a technique of reservoir probing and testing which yields results on c and ϕ that are supplementary to the conventional well test techniques (see Bodvarsson and Zais, 1981).

Fracture flow

In many types of reservoir formations, the mode of fluid transport is dominated by flow through a network of fractures rather than through a porous rock matrix. This applies in particular to geothermal reservoirs that are most frequently embedded in formations of predominately igneous origin. Quite often the global characteristics of flows through relatively dense networks of very narrow fractures can be taken to resemble Darcy type flows to such a degree that the global pressure field satisfies simple diffusion equations of the type (2) above. The above theory based on Darcy's law would for all practical purposes apply in such cases.

Frequently, however, individual fractures with apertures of the order of millimeters, or linear arrays of such fractures, domiante the flow. The above theory is then a poor approximation and the flow through single fractures will have to be considered. The pressure field theory presented by Bodvarsson (1985) would then provide a basis for reservoir modeling.

Linear arrays of fractures, or fracture ladders, as we shall call them, have, on the other hand, to be considered in the present context. Since the energy balance of many geothermal systems indicates fault zone controlled subsurface flows over very considerable distances (Bodvarsson, 1983a), it is likely that fracture ladders are present in major fracture zones where they may extend over distances of the order of tens or even hundreds of kilometers. We will attempt at presenting a very simple semi-quantitative theory for the pressure field in such structures.

The model for the fracture ladder is sketched in Figure 1 below. The basic element is a fracture space of aperture h extending over a length L along the flow and over a similar length across the flow. The individual elements are connected at the lines of offset shown in the figure. There is no reason for modeling the offsets in any detail. They simply represent lines of discontinutity where the fracture walls touch and thereby terminate the open fracture spaces. As indicated by the terminology, we take that these can be some offset between the

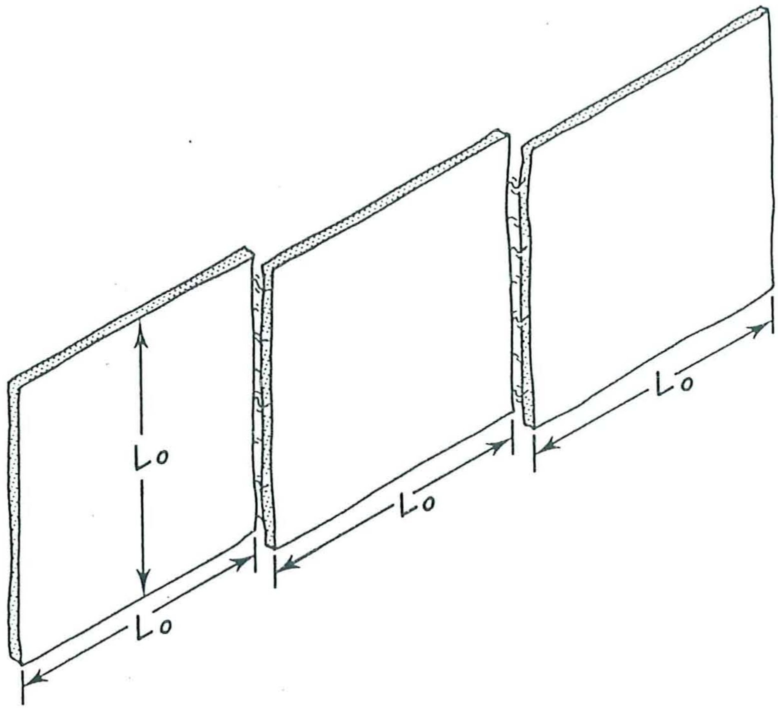

Fig. 1. The fracture ladder.

individual elements along these lines. It is not necessary to assume a uniform aperture of the elements, but we assume that the offset connection are sufficiently open to provide a global fluid conductivity for the entire ladder.

We can now establish the characteristic parameters of the ladder. Since this is a multiple-cavity system the capacitance and conductance parameters will be defined per units length both along and across the s tructure and will therefore represent specific averages.

Turning first to the capacitance, we note that the basic element is a square formed thin-box type fracture space of edge length L_0 that is embedded in a homogeneous and isotropic Hookean elastic space. No analytical expressions are available for the elastance of such models. Fairly good approximation can be worked out, but because of the semi-quantitative nature of our development here, we will resort to estimating the elastance with the help of results of Sneddon and Lowengrub (1969) for penny-shaped openings. It is quite evident that the elastance of the square fracture of edge length L_0 will not deviate substantially from the elastance of the penny-shaped fracture of diameter L_0. Let μ be the shear velocity of the rock, a good estimate for the elastance is then provided by

$$e = L_0^3/4\mu \tag{14}$$

The capacitance per unit length along and across the flow, that is, per unit area, is then

$$S = L_0/4\mu \tag{15}$$

Moreover, we take that the total flow impedance of the entire ladder is Z and the impedance per unit area is therefore Z/nL_0^2 where n is the number of elements. The specific conductance is then

$$C = nL_0^2/Z \tag{16}$$

In the case of a single ladder element of a constant aperture h,

$$C = h^3/12\nu \tag{17}$$

In other words, this is the cross-fracture integrated conductivity, that is analog to the transmisivity as defined in the literature on reservoir engineering.

On the basis of (15) and (16) follows the diffusivity along the ladder

$$a = C/\rho S = 4C\mu/\rho L_0 \tag{18}$$

Diffusion parameters

The pressure field phenomena discussed above are all of a diffusive nature and exhibit therefore a typical relaxation behavior. This is evident from the nature of the time dependence of the Green's functions given by (5) , (6) and (12). The character of the diffusive fields is best illustrated by three elementary physical parameters, viz., the skin depth d_s, the relaxation time t_r and the penetration depth d_p. A brief discussion follows of the derivation and relevance of these quantities in the case of the above models.

The skin depth is, of course, a very well known parameter that represents the diffusion distance over which the amplitude of a harmonic wave is attenuated to $(1/e) = 0.37$ of its initial value. The quantity is obtained by assuming a homogeneous one-dimensional model, inserting the wave-form $\exp[i(kx-\omega t)]$ into the basic equation and deriving the wave-number k as a function of ω on the basis of the resulting dispersion equation. Let $k = k_r+ik_i$ and we obtain then $d_s = (1/k_i)$. The real part k_r determines the phase-lag associated with the diffusion.

The relaxation time t_r is a related quantity that represents the time during which the amplitude of a stationary harmonic wave-form decreases to $(1/e) = 0.37$ of its initial value. The quantity is obtained by inserting the wave-form $\exp[-(t/t_r)+ikx]$ into the basic equation and evaluating the resulting dispersion equation.

Finally, the penetration depth d_p provides a measure of the penetration of a step-like boundary transient into a homogeneous half-space. In other words, in the present context we assume a homogeneous and isotropic Darcy type half-space with an initial pressure distribution $p = 0$. At time $t = 0$, the pressure at the boundary is suddenly raised to p_o. An elementary exercise in simple diffusion theory (equation (2)) (Carslaw and Jaeger, 1959, section 2.5) shows that the associated fluid flow into the solid is

$$q = p_o c(\pi a t)^{-\frac{1}{2}} \tag{19}$$

which we write

$$q = p_o C_o \tag{20}$$

where

$$C_o = c/d_p \tag{21}$$

is the contact conductance and

$$d_p = (\pi a t)^{\frac{1}{2}} \tag{22}$$

is the thickness of the boundary resistance. The parameter d_p defined by (22) is a measure of the penetration of the surface pressure disturbance into the solid and is therefore called the penetration depth. It is to be noted that the above definition of d_p is restricted to the simple homogeneous and isotropic Darcy type case. Other types of diffusion lead to different expressions for the parameter.

The above parameters are closely related. This is most obvious in the case of simple homogeneous and isotropic Darcy type solids. Identifying in this case the penetration time with the oscillation period we obtain

$$d_p = \pi d_s \tag{23}$$

and identifying penetration time with the relaxation time results in

$$L = 2\pi^{\frac{1}{2}} d_p \tag{24}$$

where $L = 2\pi/k$ is the wave-length associated with the wave-number k.

Omitting further details of derivation, expressions for the three parameters defined above are listed in Table I below for the three diffusion models that have been discussed, viz., the Darcy flow case, the single fracture case (Bodvarsson, 1985) and the fracture ladder. Moreover, the skin-depth and relaxation time for the free surface diffusion have been added as a fourth case in the second and fourth columns of Table I. These quantities are derived on the basis of (11) (Bodvarsson, 1984). Obviously, the skin-depth of the surface is not a well defined concept. Because of the very much longer relaxation times for the free surface, the compressibility effects are being neglected in this case. Notation is the same as has been used hitherto. It should be rieterated that the Darcy flow and the fracture ladder cases are governed by similar simple diffusion equations (2) with constant parameters, while the single fracture case leads to the more complex case (see Bodvarsson, 1985). At this time, we are unable to derive an analytical expression for the penetration depth for the single fracture or the free surface.

For later development, it is of interest to list also a modified version of the expressions in Table I. The vertically integrated specific conductance or the transmissivity C of a homogeneous Darcy type layer of thickness H is simply $C = cH$. The corresponding expression for a fracture space of uniform aperture h is given by equation (17). Expressing the parameters in cases (1) to (3) in Table I in terms of the specific conductance rather than conductivity and inserting expressions for the diffusivity a we obtain the list in Table II below. In the

Table I

Diffusion Parameters

Model	Diffusivity	Skin-depth d_s	Penetration depth d_p	Relaxation time t_r
(1) Darcy flow	$a = c/\rho s$	$(2a/\omega)^{\frac{1}{2}}$	$(\pi a t)^{\frac{1}{2}}$	$1/ak^2$
(2) Single fracture*	not defined	$[h_o\mu/2\rho R\omega(1-\sigma)]^{1/3}$	no analytical expression	$k^3/[h_o\mu/2\rho R(1-\sigma)]$
(3) Fracture ladder	$a = C/\rho S = 4C\mu/\rho L_o$	$(2a/\omega)^{\frac{1}{2}}$	$(\pi a t)^{\frac{1}{2}}$	$1/ak^2$
(4) Free liquid surface over a Darcy flow half-space**	not defined	$w/\omega = cg/\phi\omega$	no analytical expression	$1/wk = \phi L/2\pi cg$

$k = 2\pi/L$ = wave number, L = wave length

$\omega = 2\pi/\tau$ = angular frequency, τ = period

L_o = length of ladder element, $w = cg/\phi$

* Bodvarsson (1985).
** Bodvarsson (1984).

Table II

Diffusion parameters in terms of specific conductances

Model	Diffusivity	Skin-depth d_s	Penetration depth d_p	Relaxation time t_r
(1) Darcy flow	$a = C/\rho sH$	$(2C/\rho sH\omega)^{\frac{1}{2}}$	$(\pi Ct/\rho sH)^{\frac{1}{2}}$	$\rho sHL^2/4\pi^2C$
(2) Single fracture *	not defined	$[C\mu/2\rho\omega(1-\sigma)]^{1/3}$	No analytical expression	$(2\pi)^3/[C\mu/2\rho R(1-\sigma)]L^3$
(3) Fracture ladder	$a = 4C\mu/\rho L_o$	$(8C\mu/\rho L_o\omega)^{\frac{1}{2}}$	$(4\pi C\mu t/\rho L_o)^{\frac{1}{2}}$	$\rho L_oL^2/16\pi^2C\mu$

Darcy flow $C = cH$

Single fracture $C = h_o^3/12\nu$

Fracture ladder $C =$ empirical parameter

$H =$ Thickness of Darcy layer

* Bodvarsson (1985).

single fracture case, it is being assumed that $R = 12\nu/h_0^2$ (Bodvarsson, 1985) and hence $C = h_0^3/12\nu$.

Overview of reservoir test methods

Reservoir testing is an important exploration technology that has developed rapidly during the past few decades. In particular, petroleum reservoir engineering and testing are highly developed and there is a volumous literature available on these subjects (see for example, Matthews and Russel, 1967, and Earlougher, 1977). An important paper on geothermal well testing has been published by Ramey (1976). There is therefore, no reason for including a review of reservoir test methods in this report. Only a few topics of specific relevance to the subject matter of this paper will be reviewed briefly in the following.

Field techniques. To obtain some clarity as to the concepts set forth it is of interest to include the compact overview of present reservoir test procedures listed in Table III. Figure 2 supplements the table by indicating signal paths. Moreover, the period ranges of the natural drives are given in Table IV below.

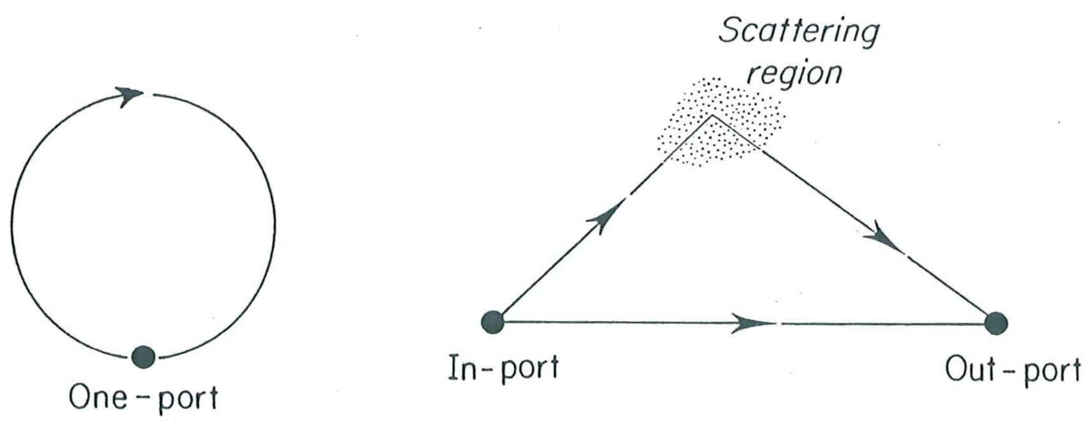

Fig. 2. Signal paths in reservoir testing.

Table III Reservoir test procedures

Driving/observation port arrangement	Signal path	Drive: Controlled	Drive: Natural	Targets
One-port Driving-point response	back scatter	Production/injection step-drive well-pressure buildup/drawdown	Oscillatory tidal/meteorological/ seismic	Diffusivities, Fluid conductivity
Multi-port transfer response	transfer & scatter	Production/injection step-drive well-interference	Oscillatory tidal/meteorological/ seismic, but scattered signal weak	
Unconventional techniques		Free liquid surface response to production		
		Hydroelastic resonance tests		Fracture elastance dimensions

Table IV

Periods of natural drives in seconds

(1)	Seismic displacement/strain	$1\text{-}10^3$
(2)	Solid earth tidal strain	$10^4\text{-}10^6$
(3)	Atmospheric pressure (barometric)	$10^4\text{-}10^6$
(4)	Precipitation load	$10^4\text{-}10^6$
(5)	Seasonal water-level variations	$10^6\text{-}10^8$

Items (2) and (3) provide the most suitable natural drives. Cases (1), (4) and (5) are possibilities that have not been given much attention and remain to be tested.

Unconventional test techniques based on present results

The development set forth above provides a basis for the discussion of a number of unconventional reservoir test techniques that will be reviewed briefly.

Natural field drive. The natural drives listed in Table IV above can be applied to carry out simple one-port driving point tests on individual boreholes that are available for such purposes. The tidal and barometric drives are of particular interest in this respect. Their frequency band is generally subinertial and observational data on uncomplicated field situations are quite easily interpreted in terms of the lumped system parameters, such as elastance, resistances and/or admittances that are relevant in individual cases. To generate sufficient data a multifrequency drive must be available.

Bodvarsson and Hanson (1981) and Bodvarsson (1983b) have discussed such reservoir test procedures based on solid earth tidal drive. It is important to remark that the former paper is devoted mainly to the case of a free liquid level in the borehole and the limitations on the tidal factor resolution imposed by this setup are considered. The results by Bodvarsson (1983b) show that this difficulty can be largely alleviated by reducing the borehole stiffness S with the help of a surface basin or a comparable arrangement. Clearly, the one-port tidal test can only provide reservoir data of a local nature. In cases where the amplitude of borehole-borehole scattered signals can be resolved there is some possibility of obtaining more global interference data on the basis of tidal tests.

The use of seismic, precipitation and seasonal water-level drives in reservoir testing has not received much attention. The theory of seismic and seasonal drive models is briefly covered by Bodvarsson (1983c). Formation pressure variations due to precipitation load pose no theoretical problems but this topic will not be perused further in this paper.

Free liquid surface responses. The presence of a free liquid surface is an important complication in the case of many water and geothermal reservoirs. As indicated above, Bodvarsson (1977, 1984) has derived a simple technique of linearizing the non-stationary boundary condition involved that is quite helpful in simplifying the theory of pressure diffusion in reservoirs with a free surface. Since this surface is a constant pressure boundary, the monitoring of its shape and position yields important data on the reservoir pressure field and provides the basis of the free surface reservoir test method that has been discussed by Bodvarsson and Zais (1981). A numerical investigation shows that at normal porosities and fluid conductivities, the free surface relaxation times (see Table I) are substantially longer than corresponding times for compressibility related phenomena. The two processes can thus be separated and treated independently as has been the procedure in this report.

Frequently there is an interest in a downward continuation of the pressure disturbance due to the free surface deformation. A convenient technique for this purpose is described in a short note by Bodvarsson (1983d).

Fracture tests based on hydroelastic resonances. In a number of field situations there is an interest in estimating dimensions of open fracture spaces that are connected to boreholes. Problems of this type occur, in particular, in hot-dry-rock technology where water/rock contact areas are of dominant interest.

Obviously, unless the fracture geometry is known there is no clear relation between the fracture dimensions and elastance. Moreover, only in the case of the penny-shaped fracture is there a simple relation between the radius and the elastance Bodvarsson (1983c). However, making certain assumptions about the geometry one can establish useful semi-quantitative relations of this type. Such relations can, for example, be established in the case of rectangular fractures that, as a matter of fact, appear to be likely field cases. The observation of data on fracture elastance therefore opens up some possibilities for estimating dimensions. Clearly, in the case of non-leaking systems the most convenient way of observing the elastance e would be to carry out a subinertial test either with the help of barometric excitation or by applying controlled pressure loading at the surface. Observing both the cavity pressure p and the associated borehole flow q will then yield data on the elastance e (see Bodvarsson, 1983b).

Difficulties are encountered in the case of leaky systems where the cavity admittance is finite. One possible avenue consists then in carrying out a special long-term leak test to determine the admittance and then proceed with the measurement of e. The procedure will have to be considered in detail in individual cases. Appreciable values of the admittance requries shorter periods and consequently dynamic testing that is based on observing the Helmholtz mode of the system. The theory of this method is discussed in detail by Bodvarsson (1983c) and the required field procedures are straight forward. A controlled square-wave injection drive combined with the monitoring of the cavity or bottom-hole pressure appears most convenient.

Observational port effects, resolution and interpretational problems

A number of problems of a varied nature are to be dealt with in reservoir testing. Below, we will briefly mention three aspects that deserve some attention.

Perturbations due to observational port capacitance. The boreholes that house the pressure sensors in well testing have a noticeable capacitance that perturbs the local formation pressure field and can therefore distort the pressure data under observation. This matter is of some concern and has been considered briefly by Bodvarsson (1981).

Test resolution. The Green's function of the Darcy flow model given by equation (6) is a typical diffusion or parabolic type impusle response that consists of a sum over exponentials with negative exponents. With the help of the development by Bodvarsson (1984) a similar form can be obtained for the free surface Green's function given in (12). Moreover, the fracture ladder is a simple diffusion model, and a similary structured Green's function can be obtained for the single fracture model as governed by the equation given by Bodvarsson (1985).

In a general sense, all interpretational work on diffusion models consists in an attempt at a separation of sums of the type (6). Because of the "similarity" of the individual terms in the series, a separation of even a few of the lowest order terms is practically not possible unless very precise observational input data are available. Since all field data are inevitably noisy, the separation task is fraud with difficulties and the overall resolution power of field test therefore very low. In other words, an infinity of different diffusion type models lead to very similar test response data and we have great difficulty in separating out the relevant model. This is best explained on the basis of the following observation.

Consider two homogeneous and isotropic Darcy flow models of different spatial dimension. One is a two-dimensional axi-symmetric case where there is a unit-step line source of strength m kg/ms, and the other one a three-dimensional point-symmetric case with a unit-step point source of strength m kg/s. The systems are in equilibrium at times $t < 0$ and both sources are activated at $t = 0$. The pressure signals emitted by the sources are observed at time t and distances r from the sources. For convenience, we introduce the Fourier numbers $F = 4at/r^2$ where a is the formation diffusivity. Moreover, as before, let c be the formation fluid conductivity. On the basis of simple diffusion theory (Carslaw and Jaeger, 1959), we find that the pressure signals are given by the following functions

axi-symmetric two-dimension	point-symmetric three-dimension	
$p(r,t) = -(m/4\pi c)Ei(-F^{-1})$,	$p(r,t) = (m/4\pi cr)erfc(F^{-\frac{1}{2}})$	(25)

where Ei is the exponential integral and erfc the complementary error functions. Using mathematical tables and plotting the two response functions over F as shown in Figure 3 we find that they are practically identical for $0 < F < 0.5$

The short-term well interference test is therefore largely "blind" with regard to the space dimensions involved. Although the value of the amplitude factor is observable, its structure depends critically on the space dimension and this data does therefore not convey any information unless strong assumptions can be made with regard to the underlying model. Considerable caution is therefore called for in the interpretation of well interference data, and it would appear that too much confidence has been placed in the applicability of the axi-symmetric two-dimensional Theis-type solution .

<u>Darcy flow layers versus fracture networks</u>. Another interpertational indeterminacy is of a very considerable practical importance. We will again consider the use of well-interference data to determine reservoir fluid conductances or transmissivities.

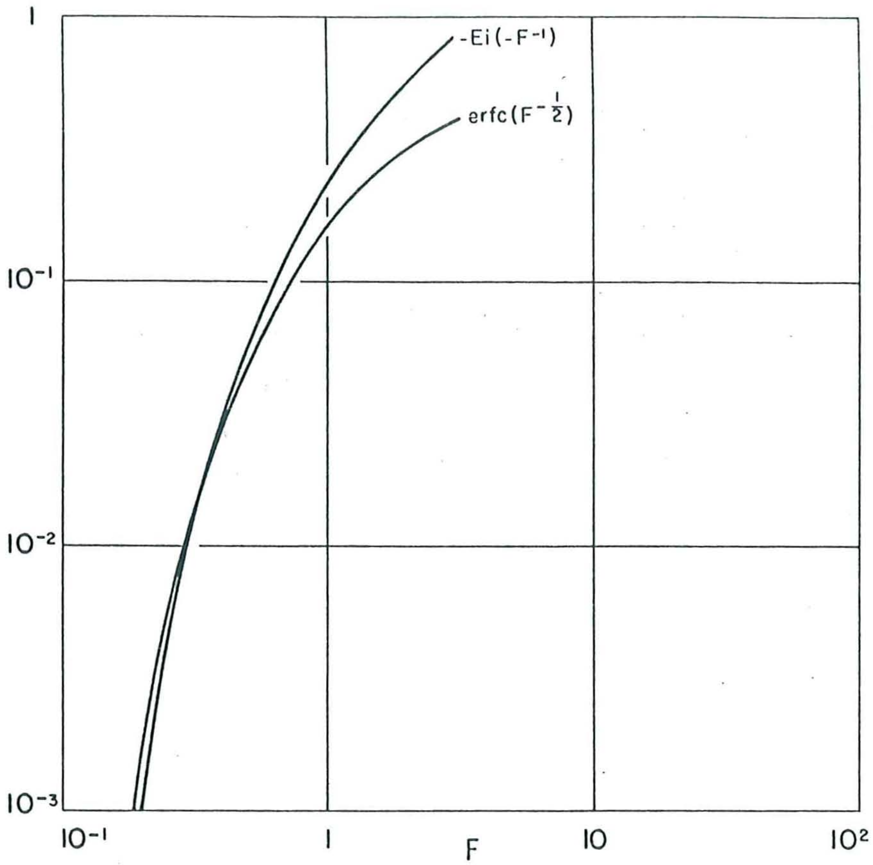

Fig. 3. Plot of Ei and erfc functions.

Consider the case of a fracture network of element length L_o as described above versus simple Darcy flow through a homogeneous and isotropic layer of thickness H. Let the subscript D refer to Darcy flow and N to the network. The ratios r_d of penetration depths with equal conductance and r_c of conductances with equal penetration depths can then be obtained from Table I and II. Using the same notation and assuming $\mu = 2x10^{10}$ Pa and $s = 5x10^{-11}$ Pa^{-1} such that $\mu s = 1$ we find that

$$r_d = (d_{pD}/d_{pN}) = (L_o/4H)^{\frac{1}{2}} \tag{26}$$

and

$$r_c = (C_D/C_N) = (H/L_o) \tag{27}$$

Since it is unlikely that natural fractures at depth can remain continuously open over large distances, we would tend to estimate L_o to be the order of say tens of meters at most. Therefore, considering cases of formation thickness 10^2 to 10^3 m, the essence of equation

(27) is that the interpretation of fracture network situations in terms of Darcy flow layers can lead to a gross overestimation of the actual specific conductance or transmissivity. Ratio of the order of 10 would appear possible.

Unfortunately, an investigation of actual field data in terms of the above conclusions has not been possible. Very few relevant data are available in the open literature. We can only mention the case of the Raft River geothermal area in southern Idaho, U.S.A. Well interference tests conducted by Narasimham and Witherspoon (1977) were interpreted in terms of specific conductance of about 6×10^{-11} m^3 (60 darcymeters) whereas production tests reveal values of about 1.5×10^{-11} m^3. This discrepancy may possibly be understood on the basis of our discussion above.

Moreover, any global directional high conductivity inhomogeneity connecting driving and observational ports will channel the pressure signal and simulate higher overall conductivities than actually is the case. Interpretation in terms of homogeneous models can then lead to a gross overestimate of the reservoir conductance. Fracture systems connecting the two observation ports would be plausible causes of this type of situation. For example, boreholes into geothermal reservoirs are preferably sited to intercept fracture zones at depth. Many such structures are highly anisotropic and may channel pressure signals in the way discussed above.

References

Bodvarsson, G., 1966, Direct iterpretation methods in applied geophysics, Geoexploration, 4:113-138.

Bodvarsson, G., 1970, Confined fluids as strain meters, J. Geophys. Res., 75(14):2711-2718.

Bodvarsson, G., 1977, Unconfined aquifer flow with a linearized free surface condition, Jokull, 27:84-87.

Bodvarsson, G., 1981, Capacitive perturbations in well interference testing, J. Eng. Ass. Iceland, 66(4):60-62.

Bodvarsson, G., 1982, Interstitial fluid pressure signal propagation along fracture ladders, J. Eng. Ass. Iceland, 67(1):12-14.

Bodvarsson, G., 1983a, Temperature/flow statistics and thermomechanics of low-temperature geothermal systems in Iceland, J. Volcanol. Geotherm., Res., 19:255-280.

Bodvarsson, G., 1983b, Resource exploration by solid earth tidal strain, J. Eng. Ass. Iceland, 68:88-97.

Bodvarsson, G., 1983c, Hydroelastic oscillations in borehole cavity systems, J. Eng. Ass. Iceland, 68:22-32.

Bodvarsson, G., 1983d, Improper continuation of geophysical continuation of geophysical potential fields, J. Eng. Ass. Iceland, 68:14-15.

Bodvarsson, G., 1984, Linearization techniques and surface operators in the theory unconfined aquifers, Water Resources Research, 20:1271-1276.

Bodvarsson, G., 1985, Hydroelastic pressure propagation in fracture spaces, J. Eng. Ass. Iceland, in press.

Bodvarsson, G., and J. M. Hanson, 1981, Geothermal reservoir testing based on signals of tidal origin, J. Eng. Ass. Iceland, 66:28-29.

Bodvarsson, G., and E. Zais, 1981, A field example of free surface testing, J. Eng. Ass. Iceland, 66:92-95.

Carslaw, H.W. and J.C. Jaeger, 1959, Conduction of heat in solids, 2nd ed., 496 pp. Oxford at the Claredon Press.

Duff, G.F.D. and D. Naylor, 1966, Differential equations of applied mathematics, John Wiley and Sons, New York, 423 p.

Earlougher, R.C. Jr., 1977, Advances in well test analysis, Society of Petroleum Engineers of AIME, New York, N.Y. and Dallas, Texas.

Lamb, H., 1932, Hydrodynamics, 6th ed., 738 pp., Dover Publications, New York, N.Y.

Mathews, C.S. and D.G. Russell, 1967, Pressure buildup and flow tests in wells, Society of Petroleum Engineers of AIME, New York, N.Y. and Dallas, Texas.

Narasimham, T.N. and P.A. Witherspoon, 1977, Reservoir evaluation tests on RRGE 1 and RRGE 2, Raft River Geothermal Project, Idaho, Lawrence Berkeley Laboratory, University of California, Berkeley, California.

Ramey, H.J. Jr., 1976, Pressure transient analysis for geothermal wells, 2nd U.N. Symposium on the Development & Use of Geothermal Resources, San Francisco.

Simon, B., 1979, Functional integration and quantum physics, Academic Press, New York.

Sneddon, I.N., and M. Lowengrub, 1969, Crack problems in the classical theory of elasticity, John Wiley & Sons, Inc., 221 pp.

Stakgold, I., 1967, Boundary value problems of mathematical physics, Volume I, The MacMillan Co. New York.

HYDROELASTIC PRESSURE PROPAGATION IN FRACTURE SPACES

Abstract

Liquid-filled fracture spaces propagate liquid pressure signals in a hyrdoelastic mode that results from an interaction of wall elasticity with the inertia of the moving liquid. The basic differential equation governing the hydroelastic pressure mode is unusual in the sense that it includes fractional powers of the Laplacian operator. Depending on frequency, the hydroelastic modes can be divided into two types of classes. First, at higher frequencies, the propagation is approximately wave-like with a phase velocity that may be from 10^{-3} to 10^{-1} times the seismic shear velocity in the surrounding rock formation. Second, at lower frequencies, the propagation is approximately diffusive with skin depths of 10^{4} to 10^{6} times the aperture of the fracture.

INTRODUCTION

Hydroelastic phenomena involve the interaction of elastic stress/strain fields with the flow/pressure fields of liquids contained in cavities, fractures and other openings in rock formations. In a recent paper (Bodvarsson, 1983), the writer has discussed hydroelastic oscillations in borehole-cavity systems, that are modeled as the dynamic interaction of a single lumped liquid mass in the borehole with the lumped elastance of a single cavity intersected by the hole. While system lumping of this type is a useful and convenient approximation, the dynamics of the underlying model does by no means cover many of the hydroelastic phenomena that may occur in natural systems. For further development of the theory, we will have to consider distributed systems, in particular, cases where the cavity consist of a fracture that has the proper aperture for the hydroelastic propagation of pressure signals. In the present paper, we will derive the basic equations for these phenomena in fracture-type structures. Before entering into the main subject a few mathematical results will be needed.

Surface operators

In the present section, we will introduce surface operators in a slightly more general context than necessary below (Bodvarsson, 1977).

Consider a rectangular coordinate system and a simply connected 2-dimension domain Σ with a piecewise smooth boundary γ embedded in the plane $z = 0$. Let B be the 3-dimensional cylindrical product domain of Σ and the interval $(0,d)$ on the z-axis. Moreover, let $\Sigma + \Gamma$ denote the boundary of B where Γ consists of the side surface of B and the end face in the $z = d$ plane. The general field point in B is $P = (x,y,z)$.

Let $u(P)$ be a function which is harmonic in B, that is,

$$\Pi u = 0 , \qquad P \text{ in } B, \qquad (1)$$

where $\Pi = -\nabla^2$ is the Laplacian operator. We assume that u(P) satisfies the Neuman type boundary condition on Γ,

$$\partial u/\partial\bar{n} = 0 \qquad\qquad P \text{ on } \Gamma \qquad\qquad (2)$$

where $\bar{n}$ is the outward normal to Γ. Since u(P) is harmonic in B and satisfies a prescribed condition on Γ, ist values in B are uniquely determined by the boundary values $u_0(S)$ on Σ where S = (x,y) is a field point on Σ. The function can be represented (Duff and Naylor, 1966) by an integral over Σ

$$u(P) = \int_\Sigma G(P,T)u_0(T)da_T, \qquad P \text{ in } B, \qquad (3)$$

where G(P,T) is the appropriate Robin function, T = (x',y') and $da_T = dx'dy'$. Evidently,

$$\lim_{z \downarrow 0} G(P,T) = \delta(S-T) \qquad (4)$$

where $\delta(S-T)$ is the 2-dimensional delta function of S centered at T.

Moreover, the various derivatives of u(P) can also be represented by integral expressions derived from (3). We are particularly interested in the negative derivative with respect to z taken at Σ, that is, at z = 0. This quantity is conveniently expressed

$$-\partial_z u\big|_{z\downarrow 0} = Lu_0 \qquad (5)$$

where L is the integral-differential operator

$$L = -\partial_z\big|_{z\downarrow 0}\int_\Sigma G(P,T), \qquad P \text{ in } B \qquad (6)$$

and the integration is with respect to T. This is a cross- surface differential operator which generates the derivative of the harmonic function u(P) across the surface Σ in terms of the values of u(P) on Σ.

The principal characteristics of L are immediately revealed by the observation that applying L twice to $u_o(S)$ we obtain because of (1) and the smoothness of u(P)

$$L^2 u_o = \partial_{zz} u(P)|_{z \downarrow 0} = \Pi_2 u_o \tag{7}$$

where

$$\Pi_2 = -(\partial_{xx} + \partial_{yy}) \tag{8}$$

and consequently

$$L = \Pi_2^{\frac{1}{2}} \tag{9}$$

that is, the operator L acts as a square root of the 2-dimensional Laplacian.

The property (9) does not determine L uniquely. There is also a dependance on d. The simplest, and in the present context the most important, case is obtained when $d \to \infty$. We can then construct the following eigenfunction representation of L.

Let $\phi_j(S)$ be the eigenfunctions and λ_j the eigenvalues of Π_2 on Σ with the Neuman type boundary condition (2) on γ, viz.,

$$\Pi_2 \phi_j = \lambda_j \phi_j \ , \qquad j = 1,2 \ . \ . \ . \ , \qquad S \text{ in } \Sigma \tag{10}$$

and

$$\partial \phi_j / \partial \bar{n} = 0 \ , \qquad S \text{ on } \gamma \tag{11}$$

Considering now the case where $d \to \infty$, it is a simple matter to show that any solution of (1) which satisfies (2) can be represented by the following eigenfunction expansion,

$$u(P) = \sum_j a_j \phi_j(S) \exp(-\lambda_j^{\frac{1}{2}} z) \tag{12}$$

where the a_j's are expansion coefficients. The Laplacian on Σ with the condition (11) has the representation

$$\Pi_2 = \int_\Sigma \sum_j \lambda_j \phi_j(S) \phi_j^*(T). \tag{13}$$

A little algebra based on (6) and (7) reveals that

$$\Pi_2^{\frac{1}{2}} = \lim_{z \downarrow 0} \int_\Sigma \sum_j \lambda_j^{\frac{1}{2}} \phi_j(S) \phi_j^*(T) \exp(-\lambda_j^{\frac{1}{2}} z), \tag{14}$$

and has an inverse

$$\Pi_2^{-\frac{1}{2}} = \lim_{z \downarrow 0} \int_\Sigma \sum_j \lambda_j^{-\frac{1}{2}} \phi_j(S) \phi_j^*(T) \exp(-\lambda_j^{\frac{1}{2}} z). \tag{15}$$

These results hold for P in B, for $d \to \infty$ only, and the integration in (13) to (15) is again with respect to T. Analog results for finite depths d are easily derived but some of the above simplicity is lost. A reflection factor has to be included in (14) and (15).

Let u(P) be a Fourier transformable function of the general field point P in a three-space. Moreover, let $F(u(P)) = \hat{u}(K)$ be its Fourier transform into K-space with the general field point $K = (k_1, k_2, k_3)$. We know then that

$$F(\Pi u) = k^2 \hat{u} \tag{16}$$

where k^2 is the square of the length of a general vector in K-space. The Laplacian is thus transformed into the simple operator k^2. From this we infer that the transform of $\Pi^{\frac{1}{2}}$ is $\pm k$ and that this relation is independent of dimension such that also

$$F(\Pi_2^{\frac{1}{2}}) = \pm k. \tag{17}$$

In the following, we will refer to operators of the type $\Pi_2^{\frac{1}{2}}$ or $\Pi_2^{3/2}$, etc., as surface operators. They are important devices to simplfy our development below.

Stiffness operators

Consider an elastic body that in the unstressed equilibrium state has a surface Σ with the general surface point S. Loading the surface with vector forces $\bar{f}(S)$ leads to the displacement $\bar{u}(S)$.

Under quite general conditions there will be a causal relation between $\bar{f}$ and $\bar{u}$ that can be expressed

$$\bar{f} = H\bar{u} \qquad \text{or} \qquad \bar{u} = H^{-1}\bar{f} \tag{18}$$

where H is an operator that enables us to determine the applied force field on the basis of the displacements. We will refer to the operator as the stiffness operator of the surface Σ.

The simplest, and in the present context, the most relevant example is the case of the homogeneous and isotropic Hookean half-space that is loaded with liquid pressure at the surface Σ. Let p(S) be the liquid pressure at the surface point S, w(S) be the resulting displacement measured perpendicular to Σ and positive into the solid, μ be the modulus of rigidity and σ be the Poisson ratio of the material.
A well known result in the theory of Hookean solids (Love, 1927) states that

$$w(S) = [(1-\sigma)/2\pi\mu]\int_\Sigma (1/r_{ST})p(T)da_T, \tag{19}$$

where T is again the surface point of integration, r_{ST} the distance between S and T and da_T is the surface element at T. Equation (19) is a convolution and a 2-dimensional Fourier transform in Σ will therefore convert it into a simple product of the transforms of $(1/r_{PT})$ and $p(T)$. Since $F(1/r_{PT}) = (1/k)$ and let $\hat{p}$ and $\hat{w}$ be the transforms of p and w respectively, we find that equation (19) transforms into the simple form

$$\hat{w}(K) = [(1-\sigma)/\mu](1/k)\hat{p}(K), \tag{20}$$

where K is the field point in transform space. Hence,

$$\hat{p} = [\mu/(1-\sigma)]k\hat{w} \tag{21}$$

and comparing with equation (17) shows that the inverse transform of (21) is

$$p(S) = [\mu/(1-\sigma)]\Pi_2^{\frac{1}{2}}w(S) \tag{22}$$

where $\Pi_2^{\frac{1}{2}} = (-\nabla_2^2)^{\frac{1}{2}}$ is a surface operator of the type defined in the preceding paragraph.

The operator on the right side of (22) is the stiffness operator that we are interested in. For the present purpose, it is, however, convenient to modify the operator in the following way.

Consider a flat fracture space embedded between two elastic half-spaces of the type defined above. The fracture is filled with a liquid that exerts a pressure on both surfaces and we assume that the fracture aperture h is everywhere so small that the pressure p can be taken to be constant over the aperture. Let p vary along the fracture causing a deformation of the surfaces bounding the fracture. We define the stiffness operator for the fracture such that

$$p(S) = Hh(S) \tag{23}$$

where H is the stiffness operator and conclude then on the basis of (22) that

$$H = [\mu/2(1-\sigma)]\Pi_2^{\frac{1}{2}} \tag{24}$$

In performing the operations involved, we assume that the independent variable is the surface point S = (x,y) in the middle plane of the fracture.

Equations for hydroelastic pressure propagation and diffusion in fracture spaces

Following the above preliminaries, we can now derive the basic equations for the hydroelastic mode of pressure propagation and diffusion in fracture spaces. The theory to be presented below is an approximation that rests on two main assumptions. First, only small amplitude motions will be considered so that all non-linear terms in the pressure and liquid velocity can be neglected. Moreover, the longitudinal scale or wavelength of the motions is assumed to be very large as compared with the aperture h of the fracture space. As a consequence, the liquid pressure p can be assumed to be constant over the aperture and the dynamic equations can then be stated in terms of the cross section average velocity vector $\bar{u}$. In other words, we are operating with essentially two-dimensional pressure and velocity fields where as stated above, the independent variable is the point S = (x,y) in the middle plane of the fracture space. Moreover, in accordance with linearity, the specific pressure loss due to frictional losses is assumed to be given by $\rho R\bar{u}$ where R is an appropriate resistance factor or operator. The structure of R will be taken up for discussion below. The fracture model that is being introduced is sketched in Figure 1 below.

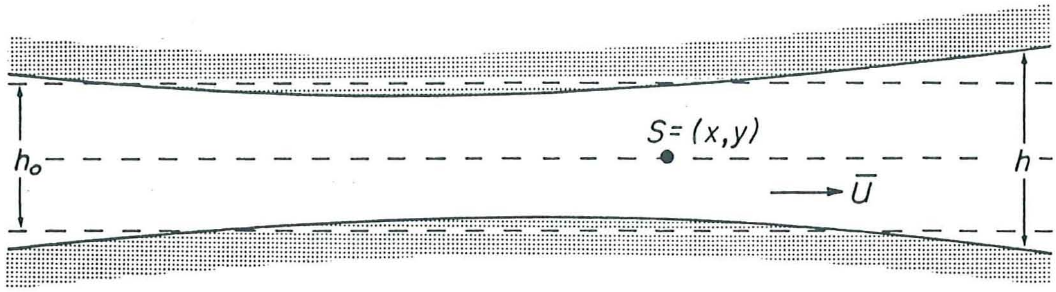

Fig. 1. Sketch of fracture model.

Let h_0 be the equilibrium aperture of the fracture that is assumed to be constant, h the local aperture of the pressurized fracture and γ be the compressibility of the liquid. Moreover, let ∇_2 be the delta operator with regard to field points S in the two-space of the fracture middle plane. Finally, to retain some generality, we introduce a source term m(S) that gives the inflow of liquid mass per unit area of the middle plane.

Referring to Figure 1, the dynamic equation is then

$$\rho\partial_t\bar{u} = -\nabla_2 p - \rho R\bar{u}, \tag{25}$$

moreover, conservation of liquid mass requires that

$$m - \rho h_o \nabla_2 \cdot \bar{u} = \rho \partial_t h + \rho \gamma h_o \partial_t p \tag{26}$$

and because of the elasticity of the bounding surfaces

$$p = Hh \tag{27}$$

These equations with appropriate boundary conditions form a complete set to determine the three unknowns p(S), $\bar{u}$(S) and h(S). It is of advantage to eliminate, for example, $\bar{u}$ and h. Inserting $\bar{u}$ from (25) and applying (27) results in an equation in the pressure

$$\rho(1 + \gamma h_o H)(\partial_{tt} + R\partial_t)p + h_o H \Pi_2 p = (\partial_t + R)Hm \tag{28}$$

where H is the stiffness operator given by equation (24). Again, it is to be emphasized that boundary conditions have to be ajoined to (28). This equation is our final result. It is characterized by the unusual surface operators $\Pi_2^{1/2}$ and $\Pi_2^{3/2}$ requiring special solution methods that have yet to be developed. Since our interest is mainly limited to propagation characteristics, we will turn our attention to fracture spaces of infinite extent where the boundary conditions are eliminated.

The resistance factor and liquid compressibility

To account for energy losses due to liquid viscosity, a term of the form R$\bar{u}$ has been introduced into the above equations (25) to (28). On a linear small amplitude theory, the term R is assumed to be a constant. There are good reasons to assume laminar flow when the Reynold number $(uh/\nu) \leq 10^3$-2×10^3 where ν is the kinematic viscosity of the liquid. Elementary laminar flow theory (Lamb, 1932) gives then a value

$$R = 12\nu/h^2 \simeq 12\nu/h_o^2 \tag{29}$$

Assuming an aperture $h_0 = 10^{-3}$m and water at 100 °C having $\nu = 3\times10^{-7}m^2/s$, the above Reynold number condition would imply a limiting velocity of 0.3 to 0.6 m/s. At most natural and many controlled conditions, this is a fairly substantial velocity that leads to a mass flow of 0.3 to 0.6 kg/s per meter fracture. On the present small amplitude theory, we may therefore with guarded confidence assume the form for R given by equation (29). It will, nevertheless, have to be underlined that very little data is available to support the above estimate of R and that further work, mainly experimental, in this field is needed.

The influence of the liquid compressibility is indicated by the magnitude of the term $\gamma h_0 H$ on the left of equation (28) as compared to unity. Since for water $\gamma = 5\times10^{-10}Pa^{-1}$ and the magnitude of the operator H is of the order of $\pi\mu/L$ where μ can be taken to be 2×10^{10} Pa and L is the wavelength, we find that for L>one meter, the term $\gamma h_0 H$ will not exceed $30h_0$. Fracture apertures of interest are, on the other hand of the order of 10^{-2} m at most, and we therefore conclude that in most cases of practical interest the term $\gamma h_0 H$ will be considerably less than unity and can therefore be neglected.

Pressure propagation

Equation (28) includes the operator $(\partial_t + R)$ which according to the arguments given in the previous paragraph can be taken to be $[\partial_t + (12\nu/h_0^2)]$. Assuming harmonic pressure fields, that is, $p \propto \exp(i\omega t)$, we conclude that the characteristics of the field are largely determined by the relative magnitude of ω and R. If $\omega >> R$, the resistance can be neglected and the pressure field will propagate as a wave-field. Disregarding liquid compressibility, equation (28) then reduces to

$$\rho \partial_{tt} p + h_o H \Pi_2 p = H \partial_t m \tag{30}$$

which is a wave-type equation. To obtain propagation characteristics, we insert the operator H from (24) and consider the homogeneous form of (30),

$$\rho \partial_{tt} p + [h_o \mu / 2(1-\sigma)] \Pi_2^{3/2} p = 0 \tag{31}$$

Inserting a wave-form $p = \exp[i(\omega t - kx)]$ where $k = (2\pi/L)$ is the wave-number into (31), and observing that in one dimension

$$\Pi_1^{\frac{1}{2}} = (-\partial_{xx})^{\frac{1}{2}} = i\partial_x \tag{32}$$

we obtain the dispersion equation

$$-\rho\omega^2 + [h_o \mu / 2(1-\sigma)] k^3 = 0 \tag{33}$$

and hence the phase-velocity

$$v = \omega/k = [h_o k \mu / 2\rho(1-\sigma)]^{\frac{1}{2}} \tag{34}$$

Taking that the liquid is water with $\rho = 10^3 kg/m^3$, the density of the rock $\rho_r = 2.700\ kg/m^3$, the Poisson ratio $\sigma = 0.25$ and the shear wave velocity is $v_s = (\mu/\rho_r)^{\frac{1}{2}}$, the above equation (34) can be restated in the simple form

$$(v/v_s) = 3.4(h_o/L)^{\frac{1}{2}} \tag{35}$$

where L is the wave-length of the hydroelastic pressure wave. A numerical evaluation of equation (35) is given in Fig. 2 below.

Pressure diffusion

Assuming that $\omega \ll R$, that is, $|\partial_t| \ll R$ in (28), and neglecting again liquid compressibility, we obtain a resistance dominated or diffusion equation for the pressure field

$$\partial_t p + (h_o/\rho R) H \Pi_2 p = (1/\rho) H m \tag{36}$$

Inserting H from (24), the homogeneous form of (36) is

$$\partial_t p + [h_o \mu / 2\rho R(1-\sigma)] \Pi_2{}^{3/2} p = 0 \tag{37}$$

which is the diffusion counterpart of the wave-equation (31). Inserting a purely attenuated wave-form $p = \exp[i\omega t - (x/d_s)]$, where d_s is the skin-depth, into (37), we obtain with the help of (32) the dispersion equation

$$i\omega - i[h_o \mu / 2\rho R(1-\sigma)](1/d_s^3) = 0 \tag{38}$$

and hence the skin-depth

$$d_s = [h_o \mu / 2\rho R \omega (1-\sigma)]^{1/3} \tag{39}$$

Inserting $R = (12\nu/h_o^2)$ into (39) and observing that the absolute viscosity of the liquid $\eta = \rho\nu$ leads to the result

$$d_s/h_o = [\mu / 24\eta\omega(1-\sigma)]^{1/3} \tag{40}$$

Assuming for the rock $\mu = 2\text{x}10^{10}$ Pa, $\sigma = 0.25$, for water at 100°C $\eta = 3\text{x}10^{-4}$ kg/ms and $\omega = 2\pi/\tau$ where τ is the period, we obtain

$$d_s/h_o = 8.4 \times 10^3 \tau^{1/3} \tag{41}$$

A numerical evaluation of the relation is given in Figure 3 below.

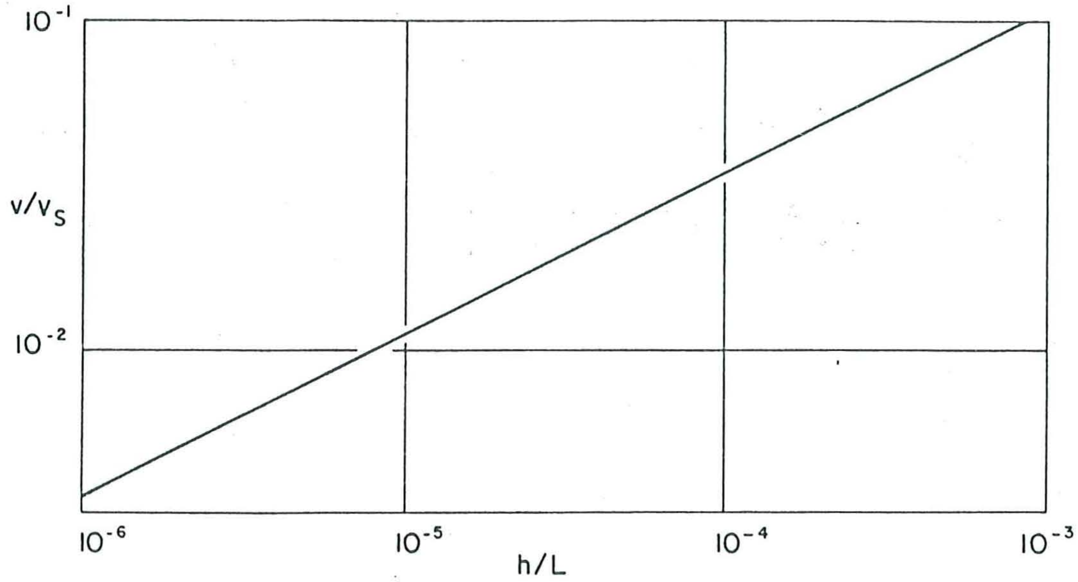

Fig. 2. Pressure propagation in water filled fractures of width h, where v_S = shear wave velocity in the rock, L = wave length, and v = phase velocity for pressure waves assuming that $\omega >> 12\nu/h^2$.

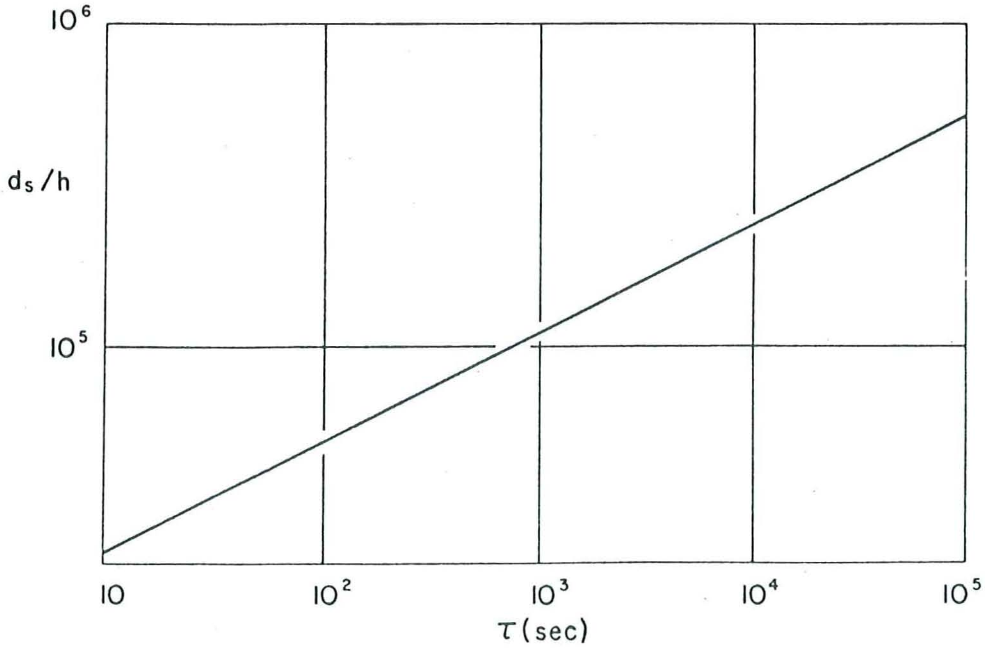

Fig. 3. Skin depth d_s for pressure diffusion in fractures of width h filled with water at 100°C and assuming that $\omega << 12\nu/h^2$.

References

Bodvarsson, G., 1977, An Equation for Gravity Waves on Deep Water, Jokull, 27:79-83.

Bodvarsson, G., 1983, Hydroelastic Oscillations in Borehole-Cavity Systems, Journ. Eng. Ass. Iceland, 68:22-32.

Duff, G.F.D. and D. Naylor, 1966, Differential Equations of Applied Mathematics, John Wiley and Sons, New York, 423 pp.

Lamb, H., 1932, Hydrodynamics, 6th ed., 738 pp., Dover Publications, New York, N.Y.

Love, A.E.H., 1927, A Treatise of the Mathematical Theory of Elasticity, New York, Dover Publications, 4th ed., 643 pp.

MECHANICS OF TWO TYPES OF HYDROELASTIC FORMATION FIELDS

Abstract

The pressure field of interstitial fluids that are contained in pores or fractures affects the elastomechanics of many types of natural formations. Moreover, the observation of interstitial fluid pressure furnishes an important method of monitoring formation strain transients. We refer to these topics as hydroelastic phenomena. The present paper discusses the mechanics of two particular types of hydroelastic models where the deformation is generated by barometric transients and solid earth tides. Inertia forces can be neglected in such cases. The first case involves the estimating of the interstitial liquid pressure in a vertically fractured slab that is affected by barometric pressure transients. The second model concerns the estimating of interstitial liquid pressure field oscillations due to solid earth tides in fault zones with a porous gouge.

Introduction

Natural formations generally contain fluids in pores, fractures and intercystalline spaces. Liquid water is the most prominent of such interstitial fluids. Most igneous formations have a very low porosity and the interaction of the interstitial water pressure with the formation elastic fields can then for most practical purposes be neglected. The situation in more fractured formations and clastic sediments, is however, different and the hydroelastic interactions, as we shall name them can then not be neglected. Moreover, the observation of fracture/pore fluid pressure is an intergral part of elastomechanical exploration work. We will therefore devote some space to the basic set of hydroelastic field equations for porous media that can be assumed to govern such processes to a satisfactory approximation.

(1) Basic equations

Consider a porous and/or fractured liquid saturated Hookean elastic formation that can be assumed to be homogeneous and isotropic on a scale much larger than the dimensions of the liquid carrying fractures. Let μ and λ be the large scale or average Lamé parameters, $\vec{u}(P)$ be the displacement vector at the field point $P = (x,y,z)$ and $p(P)$ be the interstitial liquid pressure at the field point. To establish the basic elastic field equations for this type of formation, we follow a procedure of Biot (1947) where the force field acting on the formation matrix generated by the interstitial liquid pressure is assumed to be represented by a simple body force density equal to $-\theta\nabla p$ where θ is a dimensionless parameter that is less than unity. Moreover, let $\vec{f}(P)$ be an impressed body force density. The basic large scale equations for the displacement field are then

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\nabla\nabla\cdot\vec{u}-\theta\nabla p+\vec{f} = 0 \tag{1}$$

This equation has to be adjoined by an equation for the fluid transport inside the solid. Here, we will follow a procedure of Bodvarsson (1970) and introduce the equation for the fluid pressure

$$\partial_t p+a\Pi p = -(\delta/s)\partial_t\nabla\cdot\vec{u}, \tag{2}$$

where $\Pi=-\nabla^2$, $a = C/\rho s$ is the hydraulic diffusivity, C the hydraulic conductivity, ρ the density of the fluid, s the hydraulic capacitivity and δ is the matrix coefficient introduced by Bodvarsson (1970). Equations (1) and (2) with appropriate boundary conditions constitute the basic equations that are assumed to approximately govern the hydroelastic fields of interest in the present context.

The simplified and approximate character of (1) and (2) is, however, to be noted. Without further elaboration, we will state that (1) is likely to hold mainly for formations of low fluid content as is the case for igneous formations. Moreover, the parameters θ and δ are to be taken to be purely empirical and are therefore to be adjusted to individual situations.

As may emerge a little more clearly, the pressure term in (1) has been introduced to mainly account for the pressure/elastic interaction whereas the forces on the matrix due to the flow resistance are being neglected. This approach is likely to be permissible in the case of formations of low porosity and fluid content. The introduction of the friction terms would greatly complexify the equations without much enhancing the validity for the present purpose.

There are no formal difficulties in including formation rheic effects in (1). For this purpose, we will replace the Lamé parameters in (1) by operators such that

$$(\mu+\mu'\partial_t)\nabla_2\vec{u}+(\lambda+\lambda'\partial_t+\mu+\mu'\partial_t)\nabla\nabla\cdot\vec{u}-\theta\nabla p+\vec{f} = 0, \tag{3}$$

where μ and λ are new parameters. Equation (3) thus includes elastic, rheic and hydroelastic effects.

Obviously, equations (1) to (3) are quite involved and more general types of solution are not derived too easily. However, as already emphasized, the parameter θ is generally less than unity and this permits a first order decoupling of the equations, that can then be solved by a perturbation approach. As a matter of fact, we are generally only interested in first order effects. The procedure is then to neglect the pressure term in (1) and solve for the first order displacement field $\vec{u}$. This field is then introduced as the source term in (2) that can then be solved uniquely. The matrix parameter δ is somewhat uncertain but is in most cases probably close to unity. The perturbation method outlined allows us to solve for the hydroelastic fields.

(2) <u>A specific hydroelastic model</u>

It is instructive to consider a simple specific hydroelastic model that has the advantage of allowing a specific estimate of the parameter θ. This is the case of the vertically fractured elastic half-space that is loaded by a spatially uniform pressure p_0 on the surface plane Σ. The vertical fractures are assumed to extend from the surface down to a fixed depth and be liquid saturated. The overall porosity ϕ due to the fractures is assumed to be uniform and small compared to unity. We will now derive the basic equations for this model that is sketched in Figure 1 .

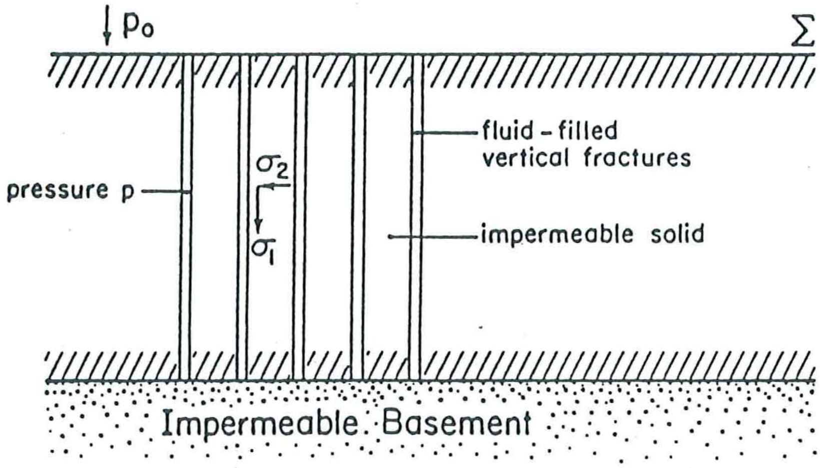

Figure 1. A specific hydroelastic model.

Let σ_1 and σ_2 be the vertical, respectively horizontal principal stresses in the solid material. Similarly, let ε_1 and ε_2 be the corresponding principal strains. Moreover, let p be the liquid pressure in the fractures. Due to the horizontal homogeneity, all quantities are functions of depth and time only. Using the notation above, we have then the modified Hooke's law for the solid material

$$\sigma_1 = (\lambda+2\mu)\varepsilon_1+2\lambda\varepsilon_2 \tag{4}$$

$$\sigma_2 = -p = \lambda\varepsilon_1+2(\lambda+\mu)\varepsilon_2 \tag{5}$$

Moreover, let q be the vertical liquid mass flow, ρ the density and c_f the compressibility of the liquid. The conservation of liquid mass requires that

$$(1/\phi)\partial_t\varepsilon_2-(1/\rho)\partial_z q = c_f\partial_t p \tag{6}$$

Equation (5) gives

$$\varepsilon_2 = -(p+\lambda\varepsilon_1)/2(\lambda+\mu), \tag{7}$$

and the liquid flow can be assumed to be governed by Darcy's law

$$q = -C\partial_z p \tag{8}$$

where C is the liquid conductivity. Let u be the vertical displacement and hence $\varepsilon_1 = \partial_z u$. Inserting these quantities in (6) yields the equation for the liquid pressure

$$[1+(1/2c_f(\lambda+\mu))]\partial_t p-(C/\rho c_f)\partial_{zz}p = -[\lambda/2c_f\phi(\lambda+\mu)]\partial_{tz}u \tag{9}$$

and (4) modifies to

$$\sigma_1 = [\lambda+2\mu-(\lambda^2/(\lambda+\mu))]\varepsilon_1-[\lambda/(\lambda+\mu)]p \tag{10}$$

If f is a vertical volume force, we obtain then the elastic equation

$$[\lambda+2\mu-(\lambda^2/(\lambda+\mu))]\partial_{zz}u-[\lambda/(\lambda+\mu)]\partial_z p+f = 0 \tag{11}$$

Equation (11) corresponds to one component of (1) and (9) is the present version of (2).

The above equations can now be simplified by assuming Poisson's relation $\lambda = \mu$ and introducing the diffusivity $a = C/\rho c_f$. Moreover, the factor $(1/2c_f(\lambda+\mu)) \ll 1$ and hence (9) simplifies finally to

$$\partial_t p - a\partial_{zz} p = -(1/4c_f\theta)\partial_{tz} u \tag{12}$$

and (11) is

$$2.5\mu\partial_{zz} u - (1/2)\partial_z p + f = 0 \tag{13}$$

Hence, in this particular case, the parameter $\theta = 1/2$.

(3) Displacement/interstitial liquid pressure relations

Observations of interstitial liquid pressure surface, displacement and tilt are complementary and the relation between these observables are of considerable interest. In the following, we will discuss this topic in a little more detail in the case of a homogeneous/isotropic permeable slab of uniform thickness h that can be assumed to be governed by equations (1) and (2). The slab is assumed to be liquid-saturated, placed on an impermeable fixed basement and for convenience we assume that the thickness h is large compared with the hydraulic skin depth at the time frequencies of main interest. The surface load is assumed to be barometric of the unit step type in time and uniform in space with an amplitude b(t). Notation is as in previous sections. Before turning to the basic equations, a few remarks have to be made as to the boundary conditions for the fluid pressure at the upper surface that will coincide with the ground water level. Several types of situations are possible.

In the case when the ground water level coincides with the free surface, the boundary pressure on the interstitial liquid would be the same as on the solid matrix, that is,

$$p = b(t), \qquad z = 0. \tag{14}$$

However, in most types of terrain, in the SW of the U.S., in particular, the ground water table is quite low and there is a thick dry surface layer that is saturated by air.

Since the ratio of air/water compressibility is of the order of 10^4 and the corresponding viscosity ratio is roughly 1/25, the compressibility by viscosity ratio is 400 and hence, assuming the same permeability, the pressure diffusivity in the wet formation will be 400 times as large as that of the dry layer. In fact, the diffusivity of the dry layer will in general be so small that barometric pressure transients will penetrate only a few tens of meters into the layer. A thick dry surface layer will therefore be largely isobaric with regard to pore/fracture pressure during normal barometric loading at the surface.

In this situation, the barometric load transmitted down to the wet formation results in an increased pore/fracture pressure that forces the interstitial water upward. Assuming small amplitudes, we have then because of the constant pressure a boundary condition of the mixed type at the ground water level (Bodvarsson, 1977), that is

$$(g/\phi)\partial_t p - C\partial_z p = 0 \tag{15}$$

where g is the acceleration of gravity.

We can now express the basic equation for this one-dimensional case of (1) and (2),

$$(\lambda+2\mu)\partial_{zz}u-\theta\partial_{z}p = 0, \tag{16}$$

and

$$\partial_{t}p-a\partial_{zz}p = -(\delta/s)\partial_{tz}u, \tag{17}$$

where p can now be taken to be the pressure in excess of hydrostatic. Moreover, in the present case, we assume causal initial conditions and taking that the barometric load is of the unit-step type of amplitude b, the boundary conditions for the displacement are

$$-(\lambda+2\mu)\partial_{z}u\Big|_{z=o} = bU(t), \tag{18}$$

where U(t) is the unit-step function, and at the base of the slab

$$u = 0, \qquad z = h \tag{19}$$

There are no formal problems in solving the above equations with either condition (14) or (15). However, since we are here only considering the general character of the hydroelastic effects, we will resort to discussing the simplest case that results when a low porosity wet formation is overlain by a high porosity dry layer such that the condition at the ground water level placed at z = o degenerates into the simple condition

$$p = 0, \qquad z = 0 \tag{20}$$

Proceeding with solving for this specific case, we assume that the porosity is sufficiently low that the above set of equations can be decoupled and solved by the perturbation procedure referred to above. The first order term u_1 is then obtained by

$$(\lambda+2\mu)\partial_{zz}u_1 = 0, \tag{21}$$

that yields

$$u_1 = Az+B \tag{22}$$

where A and B are constants for $t>0$. On the basis of the boundary conditions follows that

$$u_1 = (b/(\lambda+2\mu))(h-z)U(t), \tag{23}$$

Inserting in (15) results in

$$\partial_t p_1 - a\partial_{zz}p_1 = (\delta b/s(\lambda+2\mu))\delta(t), \tag{24}$$

and because of the large thickness h this equation has the approximate solution (Carslaw and Jaeger, 1959)

$$p_1 = (\delta b/s(\lambda+2\mu))\mathrm{erf}(z/2(at)^{1/2}) \tag{25}$$

On the basis of (16), a second order displacement is obtained from

$$(\lambda+2\mu)\partial_z u_2 - e p_1 = A \tag{26}$$

where A is again a constant. We obtain

$$u_2 = (\theta/(\lambda+2\mu))\int_0^z p_1 dz + Az \tag{27}$$

and since $u_2 = 0$ at $z = h$

$$A = -(\theta/(\lambda+2\mu)h)\int_0^h p_1 dz \tag{28}$$

which results then in

$$u_2 = -(\theta\delta b/s(\lambda+2\mu)^2)[h-z-2(at)^{1/2}(\mathrm{ierfc}(h/2(at)^{1/2})-\mathrm{ierfc}(z/2(at)^{1/2}))] \tag{29}$$

where ierfc is the integral-erfc function. The displacement at $z = 0$, that is the quantity of main interest is then to the second order

$$u|_{z=o} = (bh/(\lambda+2\mu))[1-(\theta\delta/s(\lambda+2\mu)(1-F^{-1}(\mathrm{ierfc}(F)-0.56))] \tag{30}$$

where $F = h/2(at)^{1/2}$ is the Fourier number. The extreme values of the second order approximation are then

$$\begin{aligned} t &= 0+, & u|_{z=o} &= (bh/(\lambda+2\mu))[1-(\theta\delta/s(\lambda+2\mu))] \\ t &\to \infty & u|_{z=o} &= (bh/(\lambda+2\mu)) \end{aligned} \tag{31}$$

This can be portrayed in the graph in Fig. 2. below

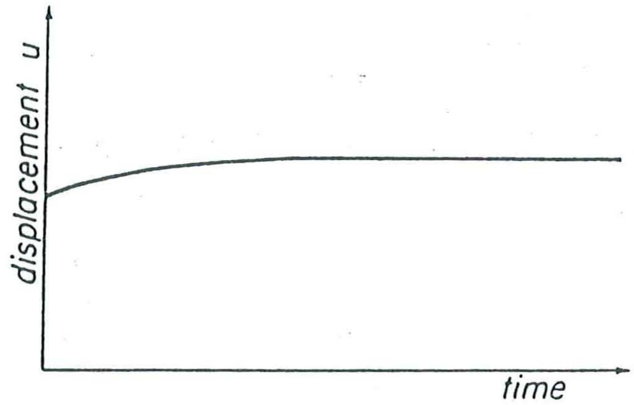

Figure 2. Displacement response of a porous fluid saturated solid.

Hence, the solid behaves as a firmoviscous or Kelvin-Vogt solid.

The first order pore/fracture pressure as given by equation (25) above is a monotonously decreasing function of time with the initial amplitude $(\delta b/s(\lambda+2\mu))$.

(4) Further development

Although the material in the previous section presents the hydroelastic processes in a very simple setting, many of the essential features emerge quite clearly. In particular, liquid saturated low-porosity formations tend to exhibit a Kelvin-Vogt type of behavior. There are no difficulties in extending our results to a more general setting. The general solution for p_1 in the one-dimensional case with the mixed type of boundary condition given by (15) is given by Carslaw and Jaeger (1959) and can be easily applied to generalize the above development. However, the algebra becomes more involved and it would appear that further work along these lines would not be warranted at this juncture. Obviously, the same applies to the study of more general three-dimensional models. There are no principal difficulties but the algebra is very involved. As of now, the main interest centers on obtaining field data.

It is, nevertheless, appropriate to indicate the procedure of applying a successive substitution or iteration technique to the hydroelastic setting. In essence, we would then reduce the displacement vector equation (1) to a one-dimensional form and insert only one displacement component in the divergence on the right of the pressure equation (2). However, it will depend on the geometry and the diffusion parameters whether any reduction of the number of independent variables in (2) can be made. This will have to be considered in the case of each individual model.

(5) Monitoring of interstitial liquid pressure

The interstitial liquid pressure has been included as a scalar p in the above equations implying that this quantity can be taken to be a localized point-function. Although there are no formal difficulties involved, it must, nevertheless, be underlined that because of observational port effects, the observation of strictly local values of p presents some difficulty.

The monitoring of p is generally carried out by placing pressure transducers in boreholes that extend to sufficient depths and serve as observation ports. Two difficulties arise under these circumstances. First, although the holes can be cased over most of their depth, most holes have an open section of a finite length and this tends to preclude the observation of strictly local values of the pressure. The values obtained are more likely to be averages over the open section. Second, all boreholes have a finite capacitance such that it is impossible to observe instantaneous values of the pressure. Again, only weighted averages over certain periods of time can be observed. The capacitance is particularly large in the case of open holes with a free liquid surface.

The capacitive effects of observation ports have been discussed briefly in a paper by Bodvarsson (1981). Although the paper addresses the problem mainly from the point of view of well interference testing, the results have relevance to a more general setting involving fluid pressure monitoring in boreholes.

(6) Exploratory relevance of interstitial fluid pressure data

The practical importance of the fluid pressure data in elastomechanical exploration and testing rests primarily with the pressure/volume-strain relation given by equation (2) above. Let H be the diffusion operator on the left of (2) and e the volume strain such that the equation can be expressed as

$$Hp = -(\delta/s)\partial_t e, \tag{32}$$

and its solution then

$$p = -(\delta/s)H^{-1}\partial_t e, \tag{33}$$

In the case of uniform strain, or very low liquid diffusivity, and at distances substantially larger than hydraulic skin depths from boundaries, equation (33) reduces to the simple relation

$$p = -(\delta/s)e \tag{34}$$

implying that the liquid pressure is proportional to the volume strain. Thus, provided the pressure data can be processed for observation port effects and the material parameter (δ/s) is known, the data give a direct information on the local value of the strain. Herein lies the main importance of the pressure observations.

The development above has been designed mainly to apply to the case of barometric forcing at the ground surface. In other words, the forcing field is vertical and generates mainly a vertical displacement. The situation with tidal forcing is different in that the forcing stress field has then to be taken to be mainly horizontal such that local displacements and tilts observed at the surface and reduced for global effects are primarily due to Poisson contraction and local inhomogeneity effects under the horizontal strain. In general, the theory of such effects tends to be more involved than the case of barometric forcing.

(7) Hydroelastic phenomena in a fault zone with a permeable gouge

An interesting tidally induced phenomenon concerns the hydroelastic effects in interstitial fault gouge liquids that are of direct relevance to the observation of interstitial fluid pressure and formation volume strain. We will elaborate briefly on this subject.

From a more general point of view, the mechanics of tidally induced hydroelastic processes in fault zones is a specific case of a more general problem that can be stated as follows. Given an infinite impermeable elastic space that includes a region B of an elastic fluid saturated porous/fracture material. The region B has a finite fluid conductivity such that the interstitial fluid flows in response to impressed pressure gradients. The entire elastic space is deformed by an impressed slowly oscillating stress field. In the general case, the elastic parameters of region B differ from the surrounding space and the region and its neighborhood will therefore be deformed relative to the space. The interstitial liquid moves in response to the impressed stress resulting in a time-dependent modification of the liquid pressure that again acts on the elastic matrix of region B and the surroundings. We are here confronted with a case of interacting elastic and liquid pressure fields. The basic equations are (1) and (2) with appropriate boundary conditions.

Since the present paper is mainly concerned with the more general physical implications of the hydroelastic effects, we will resort to briefly considering the following simplified model of a fault zone system. Consider a porous/permeable liquid saturated slab of thickness 2h that is embedded between two symmetric elastic half-spaces. The model is sketched in Figure 3 . The slab is Hookean in a generalized sense but can be inhomogeneous on a scale that is large compared to the thickness 2h. Moreover, the thickness can vary on a similar scale. The inhomogeneities are, however, restricted such that there is a well defined average thickness $2h_0$ and average material properties. The flow of the interstitial liquid is assumed to be governed by Darcy's law. Both half-spaces are homogeneous Hookean with a Poisson ratio 0.25 and a modulus of rigidity μ.

The entire system is acted on by an oscillating force field that is perpendicular to the plane of the slab and generates a principal stress of σ_0 in both half-spaces. Since the slab is inhomogeneous, the stress varies in its neighborhood but takes on the constant asymptotic value σ_0 away from the slab. Obiously, the slab will breathe under the stress and the inhomogeneities will then lead to local oscillating movements of the interstitial fluid.

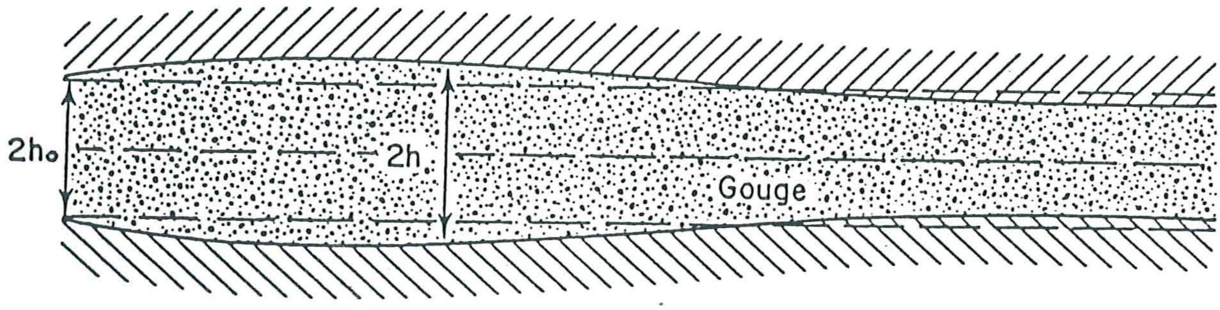

Figure 3. Model of a porous slab.

As of now, no assumptions have been made with regard to the longitudinal dimensions of the slab. For the present purpose where the interest centers on the hydroelastic effects, it is most convenient to assume that the slab is of infinite dimension. Without a significant loss of generality this eliminates boundary conditions that tend to complicate the analytical development. Because of the symmetry of the model, we have then only to consider one half of it, that is, the half-slab of thickness h welded to the adjacent half-space. The outer surface of this truncated model has to remain flat under the imposed stress. We place a rectangular coordinate system with the (x,y) plane in this surface and the z-axis vertically into the half-space.

On the truncated model the breathing of the slab leads to a displacement of the slab/half-space interface in the direction of the z-axis that conveniently is expressed $(u_0 + u)$ where u_0 is an average value that is uniform over the entire slab and $u(x,y)$ is a perturbation due to local inhomogenities in the slab material. This displacement $(u_0 + u)$ results in a linear strain of $(u_0 + u)/h$ parallel to the z-axis and it is again convenient to separate this quantity into two components such that

$$(u_0 + u)/h = w_0 + w \tag{35}$$

where w_0 is the averaged component corresponding to u_0. Associated with the strain is a resulting stress component parallel to the z-axis that we likewise write $(\sigma_0 + \sigma)$ and a pressure of the interstitial fluid of $(p_0 + p)$ where σ_0 and p_0 are again uniform over the entire slab.

Since the material of the slab is Hookean, there is a linear relation between the strain, stress and the fluid pressure that can be expressed

$$\sigma_0 + \sigma = (k(u_0 + u)/h) - \alpha(p_0 + p) \tag{36}$$

where k is a specific elastic modulus, a variant of Young's modulus that depends on the material, pore geometry and the scale of the inhomogeneities. The second parameter α is, on the other hand, a pure number that is considerably less than unity. We adjust the parameters such that

$$\sigma_0 = (ku_0/h) - \alpha p_0 \tag{37}$$

and hence

$$\sigma = (ku/h) - \alpha p \tag{38}$$

Because of the scale of the inhomogeneities, the stress in (36) and the interstitial fluid pressure are approximately constant across the slab and the equation for the pressure then reduces to a field equation in two independent variables (x,y)

$$\rho hs\partial_t(p_0 + p) - \nabla_2\cdot(ch\nabla_2 p) = \delta\rho\partial_t(u_0 + u), \tag{39}$$

where $\nabla_2 = (\partial_x, \partial_y)$ and c is the fluid conductivity. Because p_0 is uniform over the slab this term drops out of the second term on the left of (39). The expression on the right represents the effects of the volume strain where the dimensionless parameter δ, that has values of the order of unity, has been introduced to relate the volume strain to the linear strain and to account for the effects of the pressure reactance.

Finally, the displacement parallel to the z-axis, the resulting stress in the slab and the fluid pressure are related by the displacement/stress equation for the surface of the half-space. To avoid a singularity, the relation includes only the perturbation components such that

$$(\phi p - \sigma) = Hw \tag{40}$$

where $H = (\mu_c/1.5)\Pi_2^{1/2}$ is a surface operator as defined by Bodvarsson (1984) and ϕ is a dimensionless parameter that is about equal to the area porosity and has been introduced to account for the effects of the pressure on the half-space. The addition of (40) completes the system of equations.

Although the underlying model has been simplified considerably, and the resulting equations are linear, the solution of the set (37) to (40) poses a non-trivial problem. The combination of the diffusion and the half-space operator H is at the center of the difficulty. This underlines the complexity of the general tidal deformation problem we are considering in this section.

To look into the physics of the above model, we observe that when all parameters are constant, equation (39) reduces to the time integrated form

$$hsp_o = \delta w_o \tag{41}$$

and the perturbation components vanish. In the inhomogeneous case where the integrated capacitivity hs varies along the slab, and there are no external fluid sources, p_o is the source of p. Turning to a conventional perturbation case where all parameter/component perturbations are small compared to the zero order set, we obtain the following situation for the first order components. Asuming that the conductivity c is constant but the capacitivity $hs = h_o s_o + C$ where C is a perturbation, we take that now

$$h_o s_o p_o = \delta w_o \tag{42}$$

and the first order form of (39) then reduces to

$$\rho h_o s_o \partial_t p + c h_o \Pi_2 p = \delta\rho \partial_t w - \rho C \partial_t p_o, \tag{43}$$

where $\Pi_2 = (-\nabla_2^2)$.

The displacement w can be eliminated with the help of (38) combined with (40) and it is convenient to introduce the diffusivity $a = c/\rho s_o$. Equation (43) can be expressed in a final form

$$[1 - \delta(h_o s_o H + s_o k)^{-1}(\phi + \alpha)]\partial_t p + a\Pi_2 p = -(C/h_o s_o)\partial_t p_o \tag{44}$$

where the term on the right is now a source term for the pressure field.

To draw essential physical conclusions on the basis of (44) it is of main interest to consider the magnitude of the operator term in the brackets of the first term on the left. Since the operator H has a positive norm, the term $(h_o s_o H + ks_o)$ is bounded from below by ks_o. In the case of rock formations of interest in the present context, the capacitivity s_o is generally of the order of a few $10^{-11} Pa^{-1}$. Moreover, the parameter k is of the same magnitude as Young's modulus that generally is a few 10^{10}Pa. Hence, ks_o will generally be of the order unity plus, that is, in the range 2 to 3. Moreover, α is of the order of unity and based on the definition of ϕ and α, the sum of these parameters is likely to be no larger than 0.2 to 0.5. Hence, the norm of the entire operator term in the brackets of the first term on the left of (44) is in most cases below 0.2 to 0.4 and the entire brackets therefore of the order of unity. In most practical cases, equation (44) therefore is a typical diffusion equation and the related phenomena therefore dominated by the hydraulic skin-depth. This parameter gives a measure of the range of pressure diffusion under oscillatory stress. Most low-to-medium fluid conductivity cases are at tidal frequencies associated with skin-depths of the order of no more than 100 m (see Bodvarsson, 1970). We can therefore conclude that hydroelastic phenomena are not likely to greatly affect the tidal breathing of fault zones with a moderately conductive gouge. The phenomena are, however, of definite interest in the case of higher conductivities involving permeabilities well in excess of 0.1 Darcy. Skin depths can then amount to several 10^2m and are likely to substantially modify the fault breathing. In particular, the portion of the fault zone within a distance of one skin depth from the surface will be affected. The various types of tidal borehole tests discussed above provide important tools of obtaining experimental data on both stress and strain fields around fault zones of this type.

References

Biot, M.A., 1941, General theory of three-dimensional consolidations, J. of Applied Physics, 12:155-164.

Bodvarsson, G., 1970, Confined fluids as strain meters, J. Geophys. Res., 75(14):2711-2718.

Bodvarsson, G., 1977, Unconfined aquifer flow with a linearized free surface condition, Jokull, 27:84-87.

Bodvarsson, G., 1981, Capacitive perturbations in well interference testing, J. Eng. Ass. Iceland, 66(4):60-62.

Bodvarsson, G., 1984, Linearization techniques and surface operators in the theory of unconfined aquifers, Water Resources Research, 20(9):1271-1276.

Carslaw, H.W. and J.C. Jaeger, 1959, Conduction of heat in solids, 2nd ed., 496 pp. Oxford at the Claredon Press.

DYNAMICS OF BOREHOLE-FRACTURE SYSTEMS AND THE DETECTION OF FRACTURE BY ACOUSTIC TECHNIQUES

Abstract

Most geothermal systems are embedded in fracture-dominated igneous rock. Boreholes produce predominantly from fractures. The evaluation of well testing results in such reservoirs will have to be based on the dynamics of borehole-fracture systems. Acoustic pressure signals propagating down a liquid filled borehole are modified by open fractures that are transsected by the hole. The monitoring of the signals at various positions in the hole provides a method of detecting and locating fractures. In the case of a horizontal fracture that opens into the hole over the whole circumference, the smallest fracture width that can be resolved by acoustics techniques is about 0.4 millimeters.

Introduction

Well-testing is an important method of reservoir exploration. Various types of such tests furnish information on the conductive and capacitive properties of reservoir formations and can under favorable conditions provide some information on overall reservoir dimensions. The practical and theoretical aspects of well-testing have been highly developed by the petroleum industry (see e.g. Earlougher, 1977) where the emphasis has been on the testing of reservoir formations that are composed of clastic sediments where Darcy type of fluid flow prevails. The techniques have also found wide application in the water resources industry. Since geothermal reservoirs are very frequently embedded in fractured igneous formations, the interest there focuses on a somewhat different field setting requiring a modification of the underlying theory. The borehole intersecting one or more fractures in a practically impermeable formation is the relevant field model in the case of many geothermal reservoirs.

In the present paper, we consider the dynamics of a homogeneous liquid contained in a borehole-fracture system consisting of one vertical hole intersecting a single fracture at depth. The liquid reaches to the top of the hole. The theory is limited to a small amplitude excitation such that the acoustic approximation applies to the motion of the liquid. We assume that the system is being excited by controlled or natural force fields and that the liquid pressure response can be monitored at any level in the borehole. Forcing frequencies in the range 0.01-1.0 Hz that excite the basic acoustic modes of this system are of particular interest.

The setting described above is relevant to two types of field cases of practical interest. The first case relates to the use of liquid pressure signals in borehole-fracture systems to extract information on fracture location and dimension. This is frequently an important task in fractured reservoir exploration. The second case involves the application of borehole-fracture systems as seismometers. Liquid pressure signals can now be monitored with a high degree of precision such that the pressure is quite sensitive to excitation of an elastodynamical nature.

Fracture Mechanics

Before entering into the main subject, it is of interest to devote some space to the mechanics of fluid flow in fractures. Consider a plane fracture with impervious walls of uniform width h that contains a homogeneous liquid of kinematic viscosity υ. Let p be the liquid pressure and dp/dx be the pressure gradient along the x-axis that lies in the fracture plane. Assuming small velocities such that laminar conditions prevail, the fracture will then carry a mass flow q_x in the direction of the x-axis of (see e.g., Lamb, 1932)

(1) $$q_x = -(h^3/12\upsilon)dp/dx$$

Hence, the fracture has a vertically integrated conductivity or transmissivity of $h^3/12$ and thus a permeability of $h^2/12$.

Since natural liquids are slightly compressible, the fracture is a potential oscillator. The simplest situation involves the case of an inviscid, slightly compressible liquid in a fracture with rigid walls. Let the motion of the liquid be unidirectional along the x-axis. The fracture is then capable of organ-pipe type oscillations that in the acoustic approximation are governed by a simple one-dimensional wave equation in the liquid pressure p.

$$(2) \qquad \rho \partial_{tt} p - (1/c) \partial_{xx} p = K$$

where ρ is the density and c the compressibility of the liquid and K is a forcing term. In realistic situations with elastic walls, the above equation may have to be generalized to include additional terms that take the wall elasticity into account. The equation is then considerably more complex.

At this juncture, it is of interest to estimate the relative importance of the liquid compressibility and the wall elasticity. For this purpose, we consider a liquid-filled circular fracture of uniform width h and radius R with walls that are composed of a homogeneous, isotropic Hookean material. According to Sneddon and Lowengrub (1969) the elastance e of the fracture under uniform internal pressure p is

$$(3) \qquad e = dV/dp = 2R^3/\mu$$

where V is the incremental fracture volume and μ is the rigidity of the wall material. Since the fracture volume is $\pi R^2 h$, the total compressibility of the liquid is $c\pi R^2 h$. Hence, the elasticity of the walls dominates the liquid compressibility when

$$(4) \qquad 2R^3/\mu >> c\pi R^2 h$$

that is

$$(5) \qquad R/h >> c\pi\mu/2$$

Since in the case of water the compressibility is about $5x10^{-10}$ Pa^{-1} and the rigidity of common igneous rock is of the order of $2x10^{10}$ Pa, we arrive at the condition

$$(6) \qquad R/h >> 15$$

Since the width of open fractures in natural igneous formations is generally of the order of 10^{-3} m or less, it is quite clear that under most field conditions the compressibility of the liquid can be neglected. The second term on the left of equation (2) will thus have to be replaced by an appropriate term accounting for the formation elasticity. Relevant equations for this purpose have been developed by Bodvarsson (1985).

Without doubt real fractures deviate considerably from the ideal model of uniform width, simple geometry and homogeneous formation. Moreover, it is quite likely that opposite fracture walls are in contact at the tips of major asperities such that the walls are supported in these locations. For this reason, natural fractures are likely to be compartmentalized, that is, act as a series or system of interconnected open fracture spaces. Some elementary properties of ladder type of such fracture systems have been discussed by Bodvarsson (1982). Assuming that such details of fracture geometry and structure are known, it would be theoretically feasible to derive, at least, an approximate term, or series of terms, to be inserted in equation (2) to account for the wall elasticity. In the field, the problem setting is, however, such that practically nothing is known about the fractures intersected by boreholes. In fact, we are particularly interested in using observational borehole liquid pressure data to infer fracture parameters of interest. In such situations, the appropriate procedure is to construct a formal solution to the basic acoustic equation for the fracture that depends on the minimum number of unknown parameters. These quantities are then to be derived from the observational data. To obtain a sufficiently general type of solution the following procedure can be applied.

Because of the small width of fractures, we can without loss of generality assume that the local liquid pressure is constant over the width. The capacitive effects of the elastic walls can then be expressed in terms of a linear operator acting on the pressure. The operator acts in two dimensions, that is, the position coordinates in the fracture plane. Adjoining proper boundary conditions, we can because of the acoustic approximation assume that the operator is self-adjoint. In the case of a closed fracture of finite extent with a uniform internal pressure, the operator L is simply a scalar factor that depends only on the position in the fracture plane. Non-uniform pressure leads to a considerably more complex differential operator in two spatial dimension. The operator is of fractional order (Bodvarsson, 1985).

The situation can be drastically simplified in the case of compartmentalized fracture spaces. Consider the case where there are n compartments. Limiting our analysis to relative low frequencies where only the fundamental acoustic mode of each compartment is excited, the above operator L can be approximated by a n x n matrix operator that acts on the system pressure vector. The liquid pressure can be taken to be uniform in the individual compartments and the set of n compartment pressures form the system pressure n-vector. The acoustic equation then reduces to a matrix equation. Since there are quite obvious reasons for assuming that most natural fracture spaces are compartmentalized, this approximation is of particular practical interest. It represents a type of lumping of the fracture system.

Flow resistance due to liquid viscosity has not been included in the above discussion. To maintain sufficient generality, this type of attenuation will have to be considered in the basic equation. In the acoustic approximation, the appropriate procedure is to generalize the equations by including a laminar type term $b\partial_t p$ where b is a friction coefficient that is assumed to be constant over the fracture.

The fundamental acoustic equation of the fracture system is then of the form

(7) $$\rho\partial_{tt}p + b\partial_t p + Lp = K$$

where K is again an impressed forcing term. In the general case, L is a self-adjoint differential operator, while in the lumped type of approximation defined above, it is an n x n symmetric matrix operator.

A general method of solving equations of this type is to derive the eigenfuctions ϕ_j and the eigenvalues λ_j of L and express the solution to (7) in a series of the eigenfunctions. Since this is a standard basic solution method, we will refrain from discussing any detail of the procedure. The method is well described in the text by Duff and Naylor (1966).

In the present context, we are mainly interested in the driving point impulse response of the entrance to the fracture. For later purposes it is convenient to express this function in the form of its Laplace-transform with respect to time where s is the transform variable. The transform is the driving point admittance type system function A(s) that represents the input flow induced by a δ-type pressure impulse at the entrance. Using the terminology above, the result is

$$(8) \qquad A(s) = \sum_j a_j s/(\rho s^2 + bs + \lambda_j)$$

where the factors a_j are real numbers and j is the modal summation index. The series represents a sum over the normal modes of the fracture. The lowest order term is the Helmholtz mode (Elmore and Heald, 1969) where the fracture oscillates as a simple elastic cavity with an oscillating liquid mass at the entrance. In the case of the lumped approximation defined above, the series in equation (8) has a finite number of terms that is equal to the number of compartments. Equation (8) represents the desired result where the solution to equation (7) is expressed in terms of a series of the parameters a_j and λ_j.

The derivation of the physical parametes in equation (8) from observation data is an inverse problem that usually is highly underdeterminate. At the present state-of-the-art, the interest will therefore focus on the simplest situations where only the Helmholtz mode is taken into consideration. In the remainder of this paper we will therefore concentrate on this case.

<u>The Helmholtz mode of a circular fracture</u>

To look into the dynamics of the Helmholtz mode in a simple setting, we consider the circular fracture of finite extent that is sketched in Figure 1. The fracture has a radius R, is of uniform width h and opens into a borehole of radius r_o at the center. In the Helmholtz mode of this model the liquid mass in the fracture oscillates radially and the elasticity of the walls provides the restoring force. To estimate the frequency of this mode in situations where damping due to flow friction can be disregarded, we apply the energy method.

Let the angular frequency be w and the amplitude of the liquid displacement at the entrance to the fracture be unity. The amplitude of the liquid velocity at the entrance is then w. At a radial position $r > r_o$, the velocity is wr_o/r and the amplitude of the total kinetic energy of the liquid in a ringlike section of thickness dr is then

$$(9) \qquad (\rho/2)(wr_o/r)^2 2\pi rhdr = \pi\rho w^2 r_o^2 hdr/r$$

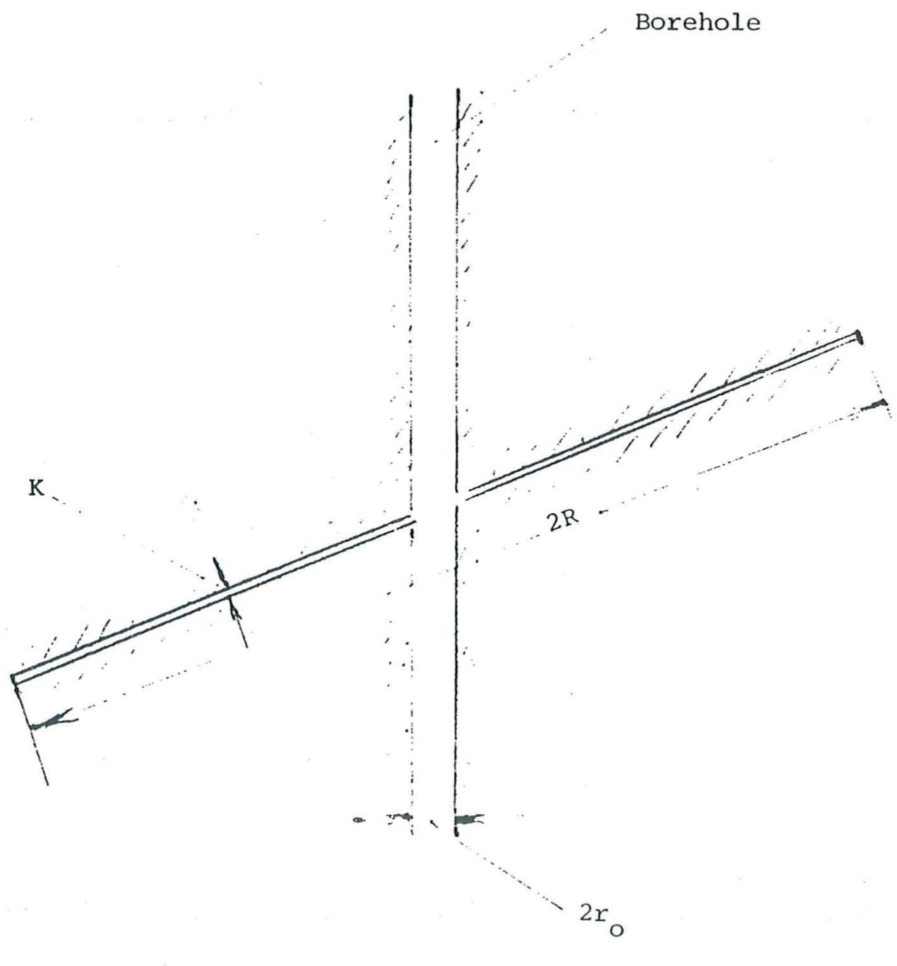

Figure 1. Sketch of an almost horizontal circular fracture transsected by a borehole.

Integrating over the whole fracture, the amplitude of the total liquid kinetic energy is

$$(10) \qquad E_k = \pi\rho w^2 h r_o^2 \int_{r_o}^{R} (dr/r) = \pi\rho w^2 h r_o^2 \ln(R/r_o)$$

The amplitude of the total elastic energy is obtained from the amplitude of the total liquid volume displacement at the entrance that is $V = 2\pi r_o h$. Let the amplitude of the liquid pressure caused by this volume increment be p such that $ep = V$ where e is the total elastance of the fracture. The amplitude of the total elastic energy is then $(1/2e)V^2$. Inserting the expression for the elastance given by equation (3), we find the amplitude of the total elastic energy

$$(11) \qquad E_e = (\mu/4R^3)(2\pi r_o h)^2$$

An approximate expression for the Helmholtz mode angular frequency w_o is obtained by equating E_k with E_e resulting in

$$(12) \qquad w_o^2 = \mu\pi h/\rho R^3 \ln(R/r_o)$$

and hence the approximate angular frequency

$$(13) \qquad w_o = [\pi\mu h/\rho R^3 \ln(R/r_o)]^{1/2}$$

This expression indicates that the radius of the fracture is the dominant parameter. To consider a plausible example, let the liquid be water such that $\rho = 10^3$ kg/m^3 and, moreover, $h = 10^{-3}$ m, $r_o = 0.1$ m and $R = 10$ m. These figures lead to a frequency $w_o/2\pi = 0.6$ Hz. A fracture of larger extent such that $R = 20$ m would have a frequency of 1.2 Hz and a large fracture of $R = 100$ m has a frequency of 0.02 Hz.

Estimates of the Helmholtz mode frequency of fractures of a more general shape can be obtained on the basis of a similar procedure albeit at the expense of a greater analytical effort. In general, it is possible to define an oscillating liquid mass m and a corresponding elastance ϵ such that the undamped Helmholtz mode frequency can be expressed as a case of a simple undamped harmonic oscillator with a simple mass m and elasticity ϵ, that is,

(14) $$w_o = 1/(m\epsilon)^{1/2}$$

To establish expressions for the parameters entering into (14), we turn again to the particular case of the circular fracture where equations (10) and (11) hold. Let f be the area of the entrance to the fracture such that in the circular case $f = 2\pi r_o h$. Equation (10) indicates that the oscillating mass can be defined

(15) $$m = \rho f \ell$$

where

(16) $$\ell = r_o \ln(R/r_o)$$

is a characteristic length that furnishes a measure of the radial dimension of the mass. Moreover, since equation (11) is obtained on the assumption that the displacement amplitude at the fracture entrance is unity, and the elastic energy of the harmonic oscillator with unity amplitude is simply $(1/2\epsilon)$, the equation indicates that

$$\epsilon = e/f^2 \tag{17}$$

Expressions (15) and (17) are applicable to fractures of a more general form than the circular model set forth above.

Having modeled the undamped Helmholtz mode of a fracture in terms of an undamped harmonic oscillator, we will now include the effects of liquid flow friction. Following the same procedure as above, consider again the case of the circular fracture of uniform width h. Let P be a position in the fracture plane and da be an area element perpendicular to the liquid flow at P. The rate of mass flow across da is q. Equation (1) indicates that the frictional force acting on the liquid element contained in the volume hda is $12\nu q da/h^3$. The inertia force acting on the same element is $(da/h)dq/dt$. Hence, in a harmonic oscillatory motion, the ratio of the frictional force to the inertia force is $12\nu/wh^2$. It is convenient to introduce the factor $\beta = 12\nu/h^2$. Consider now the lumped model for the Helmholtz mode of the fracture and let x be the displacement on the lumped model. The basic equation of motion for the pressure-displacement impulse response of the model is then

$$(md^2x/dt^2)+(m\beta dx/dt)+(f^2/e)x = K(t) \tag{18}$$

where K(t) is an impressed force. Assuming equilibrium at time $t = 0$ and applying the Laplace transformation to (18) results in

(19) $$ms^2\hat{x} + m\beta s\hat{x} + (f^2/e)\hat{x} = \hat{K}$$

where $\hat{x}$ and $\hat{K}$ are the transform of x and K. Hence,

(20) $$\hat{x} = \hat{K}/(ms^2+m\beta s+(f^2/e)) = \hat{K}/m(s^2+\beta s+\omega_o^2)$$

where w_o is the angular frequency of the oscillator. The pressure mass flow admittance of the Helmholtz mode is obtained by selecting $K(t) = \delta(t)$ and hence $\hat{K} = 1$.

(21) $$A = \rho fs\hat{x} = fs/\ell(s^2+\beta s+w_o^2)$$

This is the form of the lowest order term in the series for the admittance type system function given by (8).

The basic dynamic equations for a borehole fracture system

Consider now a vertical borehole of uniform cross section f_o and depth d_o that is filled with a homogeneous slightly compressible liquid of density ρ, combressibility c and acoustic velocity $\alpha = (\rho c)^{-1/2}$. The wall of the hole is impermeable with the exception of a fracture that opens into the hole at a depth $d < d_o$ bottom. The system is sketched in Figure 2. At this juncture, no assumptions are made as to the dimensions and permeability characteristics of the fracture. We assume, on the other hand, that at the type of excitation of interest, the fracture remains a linear type pressure-flow system that can be characterized by a causal admittance type impulse-response a(t). In other words, let the fracture be in equilibrium up to time $t = 0$. At time $t \geq 0+$ the pressure in front of the fracture varies as p(d,t). The resulting liquid flow into the

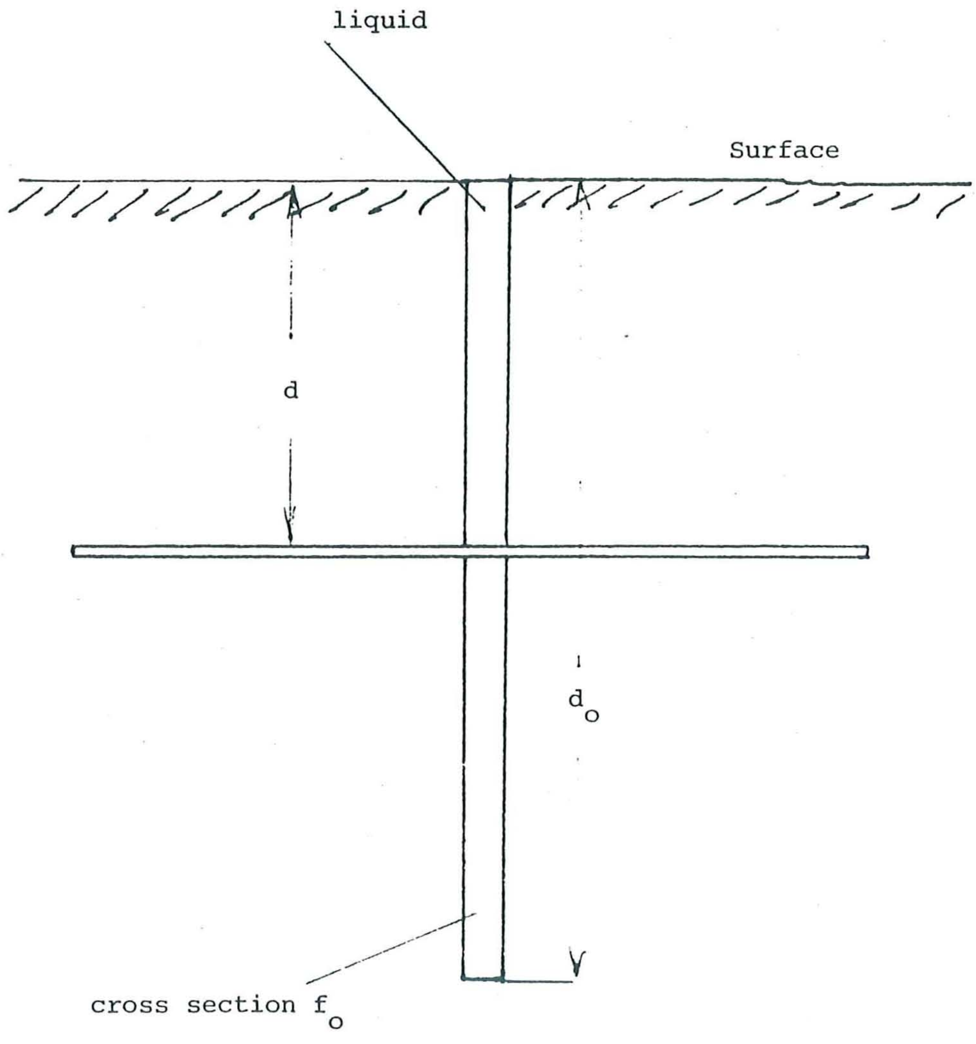

Figure 2. Sketch of a borehole-fracture system.

fracture $q_f(t)$ is then given by the convolution integral (Duff and Naylor, 1966)

$$(22) \qquad q_f(t) = \int_o^t a(t-\tau)p(d,\tau)d\tau$$

The admittance a(t) has to be derived in individual cases.

To investigate the dynamics of the borehole-fracture system we assume that the excitation is of a small amplitude such that the response of the system is in the linear acoustic range. Moreover, the frequency of the excitation is much smaller than the frequency of the lowest transversal mode of the liquid column in the hole. The column will then respond by one-dimensional organ-pipe type modes only. Since the bulk modulus of igneous rock is 20 to 40 times larger than that of water, the wall of the borehole can be considered to be rigid.

Let z be the vertical coordinate along the hole axis that is positive down with the origin at the equilibrium position of the top of the liquid column. Let u(z,t) be the liquid displacement in the organ-pipe modes of the liquid column. In the acoustic approximation, the basic dynamical equation in the displacement for the unfractured sections of the borehole is (Elmore and Heald, 1969)

$$(23) \qquad (1/\alpha^2)\partial_{tt}u-\partial_{zz}u = cb$$

where b(z,t) is an impressed force density acting on the liquid and c is the compressibility of the liquid. In the absence of liquid sources, the relation between the displacement and the liquid pressure is

$$(24) \qquad p = -(1/c)\partial_z u$$

Two boundary conditions and an initial condition have to be adjoined to equation (23). For the present purpose it is convenient to assume causal conditions, that is, vanishing initial field values and solve (23) by applying the Laplace transformation. Let $\hat{u}(z,s)$, $\hat{p}(z,s)$ and $\hat{b}(z,s)$ be the fransforms of $u(z,t)$, $p(z,t)$ and $b(z,t)$ respectively. Equations (23) and (24) transform to

$$(25) \qquad (s^2/\alpha^2)\hat{u} - D^2\hat{u} = c\hat{b}$$

and

$$(26) \qquad \hat{p} = -(1/c)D\hat{u},$$

respectively, where $D = d/dz$. Moreover, let $A(s)$ and $\hat{q}_f(s)$ be the transforms of $a(t)$ and $q_f(t)$. The convolution (22) then transforms to

$$(27) \qquad \hat{q}_f(s) = A(s)D\hat{p}(z,s)\big|_{z=d}$$

<u>Pressure signal modulation by the fracture</u>

To obtain a picture of the modulation of pressure signals by the fracture, we consider first the case where the depths d_o and d are very large but the distance of the fracture above the bottom $(d_o - d)$ is not small as compared to d_o. Let a pressure signal of finite duration propagate down the hole past the fracture. We are interested in the modulation of the signal by the fracture. For this purpose, we can assume that the borehole extends to infinity both above and below the fracture such that no boundary conditions have to be adjoined to (23). Assuming that the impressed force density $b(z,t)$ vanishes everywhere, equation (23) is then homogeneous throughout the hole except at the fracture.

In the present case, it is convenient to modify the coordinates introduced above by placing the origin $z = 0$ at the fracture. Let the amplitude of the downgoing signal be unity and the signal reflected by the fracture have the amplitude B. Moreover, let C be the amplitude of the signal that has passed by the fracture. The solution of the homogeneous form of the transformed equation (25) is then

$$(28) \qquad \hat{u}_I = \exp(-sz/\alpha) + B\exp(sz/\alpha), \qquad z < 0$$

$$(29) \qquad \hat{u}_{II} = C\exp(-sz/\alpha), \qquad z > 0$$

Since the liquid pressure has to be continuous at $z = 0$, we find on the basis of equation (26) that

$$(30) \qquad -1 + B = -C$$

Moreover, the conservation of liquid mass at the fracture then requires on the basis of (27) that

$$(31) \qquad \rho f_o s(u_I - u_{II}) = -(1/c)ADu, \qquad z=0$$

where because of (30) either u_I or u_{II} can be applied on the right. Inserting the solutions (28) and (29) results in the condition

$$(32) \qquad \rho f_o s(1+B-C) = (s/\alpha c)AC$$

Solving (30) and (31) for B and C results in

$$(33) \qquad B = A/(A_b+A)$$

$$(34) \qquad C = A/b(A_b+A)$$

where $A_b = 2f_o/\alpha$ is the acoustic pressure-flow admittance of the borehole. These equations determine the transform space modulation of the pressure signal as it passes by the fracture.

Dynamics of the borehole-fracture system with the fracture at the bottom of the hole

In continuation of the previous section, we will now turn to the dynamcis of a system consisting of a borehole of a finite depth with the fracture at the bottom, that is, the case of $d = d_o$. Having investigated the modulation of pressure signals by a fracture there really is not much loss of generality by focusing on this particular case, while the algebra is reduced considerably as compared to the more general situation where the fracture is at an arbitrary depth.

Two types of forcing of this system will be considered. First, the generation of pressure signals by the injection of a mass flow m(t) at the bottom of the hole. This would include the case of an impulsive forcing where $m(t) = \delta(t)$. Second, a forcing due to a vertical acceleration of the system that is uniform over the entire depth. In other words, the case of $b(z,t) = b(t)$. This is approximately the case where the system is excited by seismic waves propagating through the location of the borehole.

The origin of the coordinate system $z = 0$ is placed at the top of the liquid column where we assume a free liquid surface and hence a vanishing acoustic pressure. In accordance with (26), the boundary condition in transform space is then

$$(35) \qquad -(1/c)D\hat{u} = \rho g\hat{u}, \qquad z = 0$$

At the bottom $z = d_o = d$, the boundary condition is based on the conservation of liquid mass, that is, the liquid mass flow due to the acoustic signal plus the injected mass flow are equal to the mass flow entering the fracture. Analog to equations (27) and (31) above, the boundary condition in transform space is thus

$$(36) \qquad \rho f_o s\hat{u} + \hat{m} = -(1/c)A\hat{u}, \qquad z = d$$

where $\hat{m}(s)$ is the transform of $m(t)$.

Since we are mainly interested in the impulse responses, we focus on the following two cases

$$(37) \qquad m(t) = \delta(t), \quad b(t) = 0, \quad \hat{m}(s) = 1, \ \hat{b}(s) = 0$$

$$(38) \qquad m(t) = 0, \quad b(t) = \delta(t), \quad \hat{m}(s) = 0, \ \hat{b}(s) = 1$$

The task is then to solve equation (25) with the conditions (35) and (36) for these two cases. Since liquid pressure is more easily observed than the displacement, we are mainly interested in relations for the pressure. The basic solution of (25) in transform space is

$$\hat{u} = C\exp(sz/\alpha) + E\exp(-sz/\alpha) \tag{39}$$

where C and E are integration parameters to be derived by inserting the solution (39) into the boundary conditions (35) and (36). The algebra involved is elementary and can be omitted. Having derived the two parameters, we insert them into (39), apply relation (26) and obtain the following type of solution for the pressure impulse responses

$$\hat{p}(z,s) = FZB \tag{40}$$

where F is a forcing factor that in the case of the situation defined by (37) is equal to unity, while in situation (38) $F = f_o/s$. Moreover, Z is the pressure-flow impedance of the system when the borehole is infinitely deep, then according to equation (33),

$$Z = 1/((A_b/2) + A) \tag{41}$$

where $A_b = 2f_o/\alpha$ as defined in (33) and (34). The factor 1/2 results from the fact that the fracture is at the bottom of the hole. Since there is no hole below the fracture, the acoustic admittance as defined earlier has to be halved. Finally the factor B that is the signal propagator for a borehole of depth d is given by

$$B(z,s) = \frac{\exp[-s(d-z)/\alpha] - \Gamma\exp[-s(d+z)/\alpha]}{1+\Gamma\Lambda\exp[-2sd/\alpha]} \tag{42}$$

where the factors Γ and Λ are transforms of the upper and lower reflection operators given by

(43) $$\Gamma = [1+(g/\alpha s)]/[1-(g/\alpha s)]$$

(44) $$\Lambda = [(f_o/\alpha)-A]/[(f_o/\alpha)+A]$$

The response at the bottom of the hole where $z = d$ is of particular interest. From (42) follows

(45) $$B(d,s) = \frac{1 - \Gamma\exp(-2sd/\alpha)}{1 + \Gamma\Lambda\exp(-2sd/\alpha)}$$

Based on standard linear system theory (Duff and Naylor, 1966) the above relations solve the problem of general forcing $b(t)$ or $m(t)$. Let $p(z,t)$ be the inverse Laplace transform of $\hat{p}(z,s)$ corresponding to the situation (37). In the case of a general causal forcing $m(t)$, the solution for the liquid pressure is then given by

(46) $$p(z,t) = \int_o^t p(z,(t-\tau))m(\tau)d\tau$$

The same type of relation applies to the situation defined by (38) where $F = f_o/s$ and $m(\tau)$ in (46) is replaced by $b(\tau)$.

<u>Discussion</u>

The detection of the position d and estimates of the admittance functions $A(s)$ or $a(t)$ of fractures are the main fields of applications of the above development. From the theoretical point of view, at least, both fracture depth and admittance characteristics can be derived by monitoring the modulation of pressure signals as they pass by fractures. Equations (33) and (34) provide the principal relations to be applied for this purpose. Observing the shape of a pressure signal above and below a fracture, we can derive the Laplace transformation of the functions and since the borehole admittance A_b is known, we can apply equation (34) to estimate the fracture admittance A. The position of the fracture follows from the determination of the location where a modulation takes place.

Alternatively, the problem can also be approached in the time domain with the help of the inverted form of equation (31). For this purpose, it is most convenient to express this relation in terms of the acoustic velocity v(z,t) and pressure p(z,t). These quantities are related by

$$(47) \qquad \partial_t v = -(1/\rho)\partial_z p$$

and the inverted form of (31) is

$$(48) \qquad \rho f_o[v(d-,t) - v(d+,t)] = \int_o^t a(t-\tau)p(d,\tau)d\tau$$

In this formulation, the pressure is the observable and the acoustic velocity at a fixed position has to be obtained with the help of (47) as a causal integral of the acoustic pressure gradient observed at this position.

In the present context, the question arises as to the sensitivity of acoustic techniques in fracture exploration. In other words, what are the dimensions of the smallest fractures that can be located by such means. In the following, we will consider this question on the basis of the simplest fracture model, that is, the plane horizontal fracture of width h that is transected by the borehole. Relations relevant to this model have already been derived in earlier sections of this paper.

Equation (34) indicates that the ratio of the admittances $A/A_b = \alpha A/2f_o$ is the parameter that determines the acoustic sensitivity. The admittance of the circular fracture is given by equation (21) where

$$(49) \qquad f = 2\pi r_o h, \qquad \beta = 12\nu/h^2$$

and ℓ is given by equation (16).

Clearly, the admittance of a fracture increases with increasing volume elastance. Moreover, equation (21) shows that the admittance decreases with increasing frequency. Hence, at a fixed width, the maximum admittance is obtained when both elastance and inertia can be neglected. Such fractures are purely resistive, and we find on the basis of (21) and (49) that the admittance for a circular fracture of this type is then

$$(50) \qquad A_r = 2\pi h^3/12\nu \ln(R/r_o)$$

where R is the radius of the outer boundary and the subscript r refers to the purely resistive case. This result shows that A_r is a constant and the parameter C as given by equation (34) will also be a constant and the ratio A_r/A_b is thus a constant, that is,

$$(51) \qquad A_r/A_b = \pi h^3 \alpha/12 f_o \nu \ln(R/r_o)$$

To insert realistic physical parameters, we consider a borehole of a normal diameter of 0.25 m with $f_o = 0.05\ m^2$ filled with water of 50°C with $\alpha = 1.5 \times 10^3$ m/s and $\nu = 6 \times 10^{-7}\ m^2/s$. The ratio given by (51) is not very sensitive to the radius R. To satisfy the requirement of a very large elastance, we can then assume, for example, $R/r_o = 10^4$ such that the logarithmic term is almost 10. On this basis, we obtain $A_r/A_b = 1.3 \times 10^9 h^3$. Assuming for example, that the limit to sensitivity is the observation of a 10% reduction in amplitude past the fracture, that is C = 0.1 or about $A_r/A_b = 0.1$, we find that h = 0.4 mm. This would then be the smallest fracture width that could be resolved.

References

Bodvarsson, G., 1982, Interstitial fluid pressure signal propagation along fracture ladders, J. Eng. Ass. Iceland, 67, 12-14.

Duff, G.F.D. and D. Naylor, 1966, Differential equations of applied mathematics, John Wiley, New York, 423 pp.

Earlougher, R.C., Jr., 1977, Advances in well test analysis, Society of Petroleum Engineers of AIME, New York, N.Y. and Dallas, Texas.

Elmore, W.C. and M.A. Heald, 1969, Physics of waves, McGraw-Hill Book Company, New York, N.Y., 477 pp.

Lamb, H., 1932, Hydrodynamics, 6th ed., Dover Publications, New York, N.Y., 738 pp.

Sneddon, I.N. and M. Lowengrub, 1969, Crack problems in the classical theory of elasticity, John Wiley & Sons, Inc., 221 pp.