GEOHYDROLOGY OF THE LAUGARNES HYDROTHERMAL

SYSTEM IN REYKJAVIK, ICELAND

by

T. Thorsteinsson and J. Elíasson

National Energy Authority of Iceland

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Abstract.

Results of a geohydrologic survey begun in 1965 in the Laugarnes hydrothermal system in Reykjavik are presented along with an outline of the stratigraphy and structure of the Laugarnes area. The survey was conducted by recording gages and by periodic measurements of water levels in non pumping wells.

A mathematical analysis of the relation between water levels and pumping rates together with the response of the water level to changes in atmospheric pressure and to oceanic tides and earthquakes suggests the effects of volume elasticity of the Laugarnes aquifers and their water.

Theoretical considerations yield the structure of the relation between water levels and pumping rates. A linear relation is demonstrated to be valid with limited accuracy.

Analysed by the Theis nonequilibrium method the test data gave values ranging from 3.5×10^{-3} to 8.8×10^{-3} m²/sec for the coefficient of transmissivity and 3.9×10^{-5} to 3.2×10^{-4} for the coefficient of storage. The aquifers appear to be impermeably bounded on two sides, the boundaries intersecting at an angle of 60 to 90 degrees.

A decline of water level of 66.8 meters is computed for the period january 1957 to august 1969, and a further decline of 6.2 meters for the 5 year period august 1969 to august 1974, assuming 1968 to 1969 pumping rates.

1. INTRODUCTION.

The Laugarnes hydrothermal system is one of three apparently separate hydrothermal systems within a radius of 6 kilometers from the center of Reykjavik. The others are the Ellidaár hydrothermal system to the southeast and the Seltjarnarnes hydrothermal system to the west.

Exploitation of hot ground water by wells in the Laugarnes system increased rapidly from 1958 when about 30 liters per second flowed freely from a few shallow wells, until 1969, when a maximum of 330 liters per second was pumped from 11 supply wells with depths up to 2198 meters.

An investigation on the response of the piezometric surface in the area to increased pumping was begun in 1965 and continued through 1969. The investigation was conducted by automatic water stage recorders and by periodic measurements of water levels in nonpumping observation wells. It was initiated by the Department of Natural Heat of the National Energy Authority of Iceland in close cooperation with the Reykjavik Municipal District Heating Service.

2. WELLS.

The exploitation of hot ground water in the Laugarnes area was begun in 1928-1930 by the drilling of 14 small diameter wells near the Thvottalaugar hot spring (Fig. 1). The depth of the deepest well was 246 meters and together they flowed 15-20 liters per second at a temperature of 95°C, as compared to 5-10 liters per second previously issuing from the spring.

Drilling was resumed, first in 1940 by the drilling of two wells 650 and 760 m in depth at Thvottalaugar and at Raudará, and again in 1956-1959 by the drilling of 16 wells, 260-696 m deep, 1-2 km west of the Thvottalaugar wells. The aggregate flow from these wells in 1959 was about 60 liters per second, 90-98°C.

During the final drilling phase, 1959-1963, 22 wells were drilled by the rotary method to depths of 650-2198 meters. The individual wells' flowrates were 1 to more than 50 liters

per second at their times of completion and their shut in pressure 2.5 kg/cm² (Well Gl5, oct.1962). Since the completion of this final drilling program in the Laugarnes area, 11 wells, 1025-1647 m in depth, have been drilled in the Ellidaar area, 2 wells, 854 m and 1282 m in the Seltjarnarnes area and one well in Kopavogur near the Ellidaar well field.

The wells are of the open hole type. Casing is cemented in place to a depth required to prevent collapse of unconsolidated shallow formations and exclude surface waters and the hole left open below the casing point. Construction dimension of the 11 supply wells and principal observation wells are given in Tables 1 and 2.

Wells in the Laugavegur and Ellidaár areas are numbered according to type of drilling rig and time of completion. Earlier wells, drilled with calyx type rigs using steel shots or diamond bits, are designated by the prefix H and numbers 1 to 41. Wells completed after 1958 by the rotary method have the prefix G and are numbered 1 to 22 in the Laugarnes area and 23 to 32 in the Ellidaár area. The Seltjarnarnes wells and the Kópavogur well are numbered S1, S2 and K1. Figure 1 gives the location of the three hydrothermal systems while location of wells in the Laugarnes system is given on Figure 8.

3. GEOLOGY.

Reykjavík is located near the southern end of the Tertiary plateau basalt series of western Iceland, bounded to the east and south by the Reykjanes quaternary volcanics. The Laugarnes area located in the northeast central part of Reykjavík, is covered by up to 30 m thick coarse grained olivin basalt lava flows resting on interglacial sediments up to 60 m in thickness. Underneath the sediments basalt flows alternate with pyroclastics and sediments to a depth of at least 2198 meters. Down to a depth of about 1250 meters the intercalations of pyroclastics and sediments are relatively thick and many of the flow series exhibit pillow structures indicating subaquaous environment of formation. Below 1250 meters the intercalations are thinner and pillow structures are mostly absent.

TABLE 1.

Construction dimensions of supply wells.

Well No.	Year completed	Elevation, meters above mean sea level	Depth of well meters	Diameter of well millimeters	Depth of casing meters	Pump yield nov.3, 1969 Liters per second	Temperature of water degrees centigrade	Depth to pump meters	Production horiz. Aquifers A, B or C.
G 4	1959	15.48	2198	222	69	8.2	135	93	АВС
G 5	1959	15.07	741	222	68	49.1	130	110	A B
G 9	1959	27.06	862	222	90	32.4	128	116	A B
G 10	1959	15.87	1306	222	92	15.1	130	109	A B
G 11	1962	25.72	928	222	112	32.8	130	116	В
G 15	1962	24.72	1014	222	112	23.3	126	110	A B
G 17	1963	21.59	634	222	93	15.2	122	109	A B ?
G 19	1963	28.09	1239	222	79	39.1	128	116	A B
G 20	1963	26.11	764	222	87	47.0	129	111	A B
G 21	1963	24.74	978	222	112	44.2	129	116	A B
H 27	1959	14.98	403	164	31	14.7	115	109	A

TABLE 2.

Construction dimensions of observation wells.

Well No.	Year completed	Elevation meters above mean sea level	Depth of well meters	Diameter of well millimeters	Depth of casing meters	
G 1	1962	12.04	1067	222	70	
G 2	1958	20.86	650	222	30	
G 3	1958	27.03	732	222	71	
G 6	1959	27.63	765	222	99	
G 7	1959	16.90	752	222	94	
G 8	1960	11.01	1397	222	91	
G 12	1962	17.74	1105	311	94	
			1362	222		
G 13	1962	17.10	975	311	100	
			1463	222		
G 14	1962	4.28	1026	222	101	
G 16	1962	16.78	1300	222	256	
G 22	1963	30.36	1583	222	83	
G 25	1968	29.5	1647	222	79	
G 32	1969	42.0	1359	222	100	
Н 16	1943	12.36	770	152	17	
н 18	1956	8.42	697	75	19	
Н 29	1959	19.82	249	164	33	
Н 32	1961		606	101	32	
K 1	1969		1504	222	100	
S 1	1967	6.0	1282	1 58	18	

The flow series below the topmost olivin basalt and interglacial sediments have been referred to the Tertiary. The stratigraphy of the topmost 1250 meters may however indicate a relatively young age for that part of the section.

The Tertiary strata in the Laugarnes area appear to dip 3 to 12 degrees to the southeast, the larger dips being observed between wells G3, G2 and G6 in the western part of the area where key beds also appear at an elevation of 25 to 125 meters above similar beds in the central and eastern part.

The observed dips are comparable in magnitude and direction to those of outcropping strata, usually referred to the Tertiary at Vatnagardar, 2 1/2 kilometers to the east, and at Gufunes, 4 kilometers to the east, and still farther east at Olfarsfell, where a number of northeast striking dip slip faults with downthrow towards the northwest, are visible. The Laugarnes strata occur at 250 to 300 meters lower elevation in wells in the Ellidaar area, but have not been identified in wells S1 and S2 on Seltjarnarnes. Figure 2 gives a schematic N W - S E cross section through the Laugarnes area.

The strata cropping out at Vatnagardar are believed to be extruded subaquously. They are found in wells in the Laugarnes area to depths of 250 to 350 meters and to a depth of 425 meters in wells in the Ellidaár area. They appear to be related to a north-south trending oblong intrusive mass proposed by Einarsson (1954) as a result of his "Survey of Gravity in Iceland". Sections of coarse grained basalt encountered below the depth of 1200 meters in well G25 and from 600 to 1359 meters in well G32 may also be related to it.

Although the drilling area is too limited in areal extent to reveal a detailed structural picture of the three hydrothermal systems, existing data indicate a disruption of the continuity of the Laugarnes strata on two sides. Towards the southwest by a northwest striking fault system suspected from the attitude of key beds in wells G2, G3 and G6, and towards the east by the Vatnagardar intrusive and perhaps a system of northeast striking dip slip, strike faults.

4. HOT WATER EXPLOITATION.

4.1. Aquifers.

A study of the geological section and of temperature logs and hydrographs from observation wells has revealed three major, at least locally separate aquifers designated A, B and C in descending order in Fig. 2. Aquifer A with water of 110-120°C extends from 250 to 650 meters, aquifer B with water of 135°C from 730-1250 meters and aquifer C with water temperature of 146°C, below 2150 meters. Tuffs and sediments act as aquicludes between the aquifers while scoracious and fractured contacts between individual lava flows are permeable. Because each lava flow lenses out between overlying and underlying flows, the permeable zones within each aquifer are not continous but may merge with those of adjacent flows. Aquifer A is penetrated by all the supply wells in the area, aquifer B by all but wells G17 and H27 and aquifer C by well G4 only. The estimated percentages of the total withdrawal of water supplied by each aquifer are 18, 80 and 2 per cent, in the same order.

4.2. Withdrawal of water.

Up to the year 1960, when deep well turbine pumps were first installed, withdrawal of water from the wells was by flow on the head. Since 1967, however, it has been exclusively by deep well turbine pumps from 11 supply wells. Prior to 1962, flow rates were estimated from periodic flow measurements but have since then been metered.

Withdrawal rates were relatively uniform during the period 1957-1962, when withdrawal was predominantly by flow on the head but those subsequent to 1962 vary according to seasonal demand, being about three times as heavy in the winter season, october through march, than in the warmer season, april through september. During the period october 1968 through september 1969 monthly withdrawal ranged from 0.736 gigaliters (1 gigaliter = 10⁶ m³) in december 1968 to 0.241 gigaliters in august 1969 while total withdrawal during the period amounted to 6.361 gigaliters.

Figure 3 gives the metered rates of withdrawal of water from the Laugarnes well field for the period 1965 to 1969 and Table 4 gives withdrawal rates in liters per second from 1957 to 1969 averaged for the light and heavy pumping seasons.

4.3. Fluctuations of water level.

From the beginning of this investigation in 1965 water levels in 60 observation wells have been measured periodically and a total of 27 wells has since 1967 been equipped with water level recorders for various lengths of time.

Fluctuations of water levels in the observation wells reflect variations in pressure heads within the aquifers penetrated by the wells. Their major cause is variation in pumping rate from supply wells while minor causes are oceanic tides, changes in atmospheric pressure and occasionally earthquakes.

Fluctuations of water levels due to pumping in the Laugarnes well field have not been observed in shallow wells in the immediate area nor in deep wells in the Ellidaar and Seltjarnarnes hydrothermal systems indicating artesian nature of the aquifers and impermeable boundaries between the three systems.

5. INVESTIGATIONS OF THE RESERVOIR MECHANISM.

5.1. Relation between pumping rates and aquifer pressure.

5.1.1. Theoretical considerations.

The scope of the theoretical considerations is to find "a priori" a relationship between the water levels in the observation wells, and the pumping rates. To do so, we have the basic equations of hydrodynamics, such as the Euler equation, which we combine with Darcy's law, and the equation of continuity. In the mathematical treatment of ground water problems one usually assumes plane flow, that is, vertical velocities are assumed small in comparison with horizontal velocities. If we furthermore assume the water level fluctua-

tions to be small compared with the total depth of the aquifer, we have the same aquifer pressure in each vertical line,
and this pressure, which we shall call h, is completely represented by the water level, and we arrive to the following
general equation,

L
$$(\frac{\partial^2 h}{\partial t^2}, \frac{\partial h}{\partial t}, h) = \text{divgrad } h$$
 (1)

where L means some linear function. The second derivative in (1) represents the acceleration of the water, the first derivative the damping ("storage effect") and h itself the inflow term. In dealing with equations of this kind it is convenient to use the substitutions:

$$x = \frac{r}{D} \qquad \phi = \frac{h}{D} \qquad (2)$$

The following symbols are used:

r - distance from well

t - time

g - acceleration of gravity

- S storage coefficient, i.e. the quantity of water yielded by $1/m^2$ of the aquifer as a result of 1 m drawdown (S = $m^3/m^2/m$)
- T coefficient of transmissivity, equal to Darcy's coefficient of permeability times D
- D total depth of aquifer

To/D - coefficient of proportionality between inflow and ϕ

Using the substitutions (2) and (3), the equation (1) becomes

$$\frac{S}{gD} \cdot \frac{\partial^2 \phi}{\partial t^2} + (\frac{S}{T} + \frac{T_0}{gD^3}) \cdot \frac{\partial \phi}{\partial t} + \frac{T_0}{TD} \cdot \phi = \frac{1}{D^2} (\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{x} \cdot \frac{\partial \phi}{Dx})$$
 (4)

This is a linear second order partial differential equation of the hyperbolic type. In deriving it the following assumptions are used:

- 1. Axisymmetric flow towards a single well.
- 2. All vertical velocity components are negligible.

- 3. Eventual water depth variations small compared with the total depth of the aquifer.
- 4. Inflow proportional to the drawdown.

We see at once that the characteristic directions, and consequently the travel speed of an initial disturbance, is given by

$$C = \sqrt{\frac{gD}{S}}$$

In our case, when the total depth of the aquifer is of the order 1 km, this is very great speed. We can therefore assume that all pressure variations are instantaneously transmitted to the entire system. The acceleration term can therefore be omitted altogether, unless where its presence is necessary to fulfill the initial value boundary condition.

This leaves us with an equation, which for most practical purposes is of the parabolic type. That is, the damping term and the inflow term predominate the solution.

The coefficient of the inflow term is the ratio of the inflow leakage coefficient and the coefficient of transmissivity. Geological evidence shows that inflow into the aquifer must be small, due to the layers of low permeability overlying the aquifer formation (Chapter 4). On the other hand, if the inflow is zero, the drawdown h will grow without limits, so the inflow term has its effect upon the solution, however small it may be. It assures the existance of a stable solution of (4) at infinity.

The damping term coefficient, contains two terms, damping due to inflow and damping due to the storage capacity of the aquifer, it is theoretically possible to distinguish between the effects of these two terms as the inflow appears elsewhere in the equation, but it is highly unlikely that this is practically possible, because the inflow term is usually very small compared with the storage effect. On the other hand it is entirely impossible to distinguish between the storage effect of an eventual free surface, and the storage effect due to the elastic deformation of the aquifer, as both are contained in the single parameter S.

51.2. Solution of the differential equation.

In order to find a solution to the equ. (4), we must specify appropriate boundary conditions. To do so, a physical model of the aquifer must be established. The only effect the model has on the differential equation itself, is if it makes the inflow term zero, or if it makes both the inflow and the damping term zero.

In selecting a model we have already made use of the linearity of (4), by considering only flow towards one well. Solution for pumping from more wells can be found by simple addition.

We have also used that the inflow (if any) is proportional to the drawdown and depth variations (if any) are small compared with the total depth of the aquifer.

We now make the following assumption about our model: The aquifer is a semi-infinite homogeneous formation of constant thickness. The pumping starts from rest and continues at a given rate that varies with time. Mathematically this implies the following:

1.
$$t \le 0$$
, $\phi = 0$
2. $r = a$, $\frac{\partial \phi}{\partial x} = \phi'(t)$ given for all t
3. $r + \infty'$ $\phi \to 0$

We have not specified if the storage effect is due to elastic deformation of the aquifer material, or depth variations within the aquifer (unconfined flow). But as already stated no stable solution exists, except when inflow is present. It is therefore convenient to deal with the zero inflow case separately, especially as this case coincides with the classical theory of elastic aquifers, and this is done in the next section, 5.2.

The complete solution to (4) is the integral
$$\phi = \int_{-\infty}^{\infty} \phi'(s) \cdot G(x, t - s) ds$$

$$G(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp(i\omega t) \frac{H_1(\lambda x)}{H_1'(\lambda a')} \cdot \frac{d\omega}{\lambda}$$

$$H_1(z) \text{ means the Hankel function of zero order 1. kind,}$$

of the complex variable z and $H_1'(z) = d H_1/dz$

$$\lambda_{2} = \lambda_{1} + i\lambda_{2}, \qquad \lambda_{2} > 0$$

$$\lambda^{2} = \frac{\omega^{2}D}{Sg} - \frac{T}{T} - \frac{i\omega D^{2}}{ST} - \frac{i\omega To}{gD}, \quad i = \sqrt{-1}, \quad a' = a/D$$

s parameter (time), ω parameter (time) $^{+}$ 1

For constant pumping rate $\phi'(s) = const$ it may be shown that for $t \to \infty$ ϕ approaches

$$\phi(x) = \phi_0 \cdot H_1(\lambda x) \qquad \lambda = i \cdot \sqrt{\frac{T_0}{T}}$$

and by using the modified Bessel function of zero order second kind,

$$K_{O}(x) = \frac{\pi}{2} \cdot iH_{1}(ix)$$

$$\phi(x) = \phi_{O} \cdot K_{O}(\sqrt{\frac{T_{O}}{T}} \cdot x)$$
(6)

where ϕ_0 is a constant, given by boundary condition 2.

It is beyond the scope of this paper to deal with the detailed mathematical investigation of (5). We only quote, that by letting a tend to zero, when keeping a \cdot ϕ (t) constant, then we arrive to the following expression for ϕ

$$\phi(x,t) = -\frac{a'}{2} \int_{0}^{\infty} (\phi'(t-\alpha \cdot x \cdot e^{u})) + \phi'(t-\alpha \cdot x \cdot e^{-u}) \cdot \exp(-\beta \cdot x \cdot \cosh u) \cdot du$$

$$\alpha = \frac{1}{2} \left(\frac{D^{2}}{S \cdot \sqrt{T_{0}T}} + \frac{\sqrt{T_{0}T}}{Dg} \right) \sqrt{\frac{D^{2}}{2 \cdot S \cdot \sqrt{T_{0}T}}} \qquad \beta = \sqrt{\frac{T_{0}}{T}}$$

5.1.3. Processing of the observed data.

Equation (7) is unsuitable to investigation of the reservoir mechanism, as it involves a combined function of x and t. It is equivalent to:

$$h(x,t) = -\frac{1}{2 \cdot \pi \cdot T} \int_{0}^{\infty} Q(t-\tau) \cdot f(x,\tau) \cdot d\tau \qquad (8)$$

where Q is the pumping rate and f(x,T) a known function, if T and To and S are known.

It can be expected, for geological reasons, that deviations from the calculated h values will be observed, as a result of the inhomogenity of the aquifer. All such deviations

make it difficult to determine the reservoir coefficients. It is also of great importance if we are able to use simple formulae, in the investigation of the reservoir mechanism, as simple expressions can also be used for operational purposes.

We have therefore investigated the possibility of using

$$h(x,t) = -\frac{1}{2 \cdot \pi \cdot T} \quad f_{\Gamma}(x) \int_{0}^{\infty} Q(t-\tau) \cdot f(\tau) d\tau \quad (9)$$

instead of (8). For the function $f(\tau)$ we have chosen

$$f(\tau) = \theta \cdot \exp(-\theta\tau)$$

and by denoting:

$$q(t) = \int_{0}^{\infty} Q(t-\tau) \cdot \theta \cdot \exp(-\theta\tau) d\tau$$
 (10)

we get:

$$h(x,t) = -\frac{f(x)}{2\pi T} \cdot q(t)$$
 (11)

It is easy to check, whether (11) can be used or not, because the q's can be calculated once and for all without finding T and To.

The validity of (11) was checked on water level records from 7 drillholes. An almost continuous record from sept. 1965 to june 1969 was available. The calculations were carried out on an IBM 1620 II computer, using a standard program for stepwise multiple regression. Various θ values were tried, but best results were obtained with $\theta = 0.156.10^{-6} \text{sec}^{-1}$.

The regression procedure, consisted of finding the Ho's and Y's of the formula

$$H_{i} = H_{oi} - \sum_{j=1}^{J} Y_{i,j} \cdot q_{j}$$
 (12)

There are 12 wells within the area that have been used from time to another. The analysis was based on the average monthly pumping rates from these wells, so J should be 12. G07 had been used for only 3 months and was let out of the picture. G09 and G21 are only 50 meters apart, so they are treated as one, 921, and G11 were found to be almost proportional to 921, when q_j q_{j+1} are thus proportional, we have

$$q_{j+1} = q_0 + \alpha \cdot q_j(t)$$

and the corresponding terms in (12) become $Y_{i,j} \cdot q_{j} + Y_{i,j+1} \cdot q_{j+1} = Y_{i,j} \cdot q_{j}$

$$+ Y_{i,j+1} \cdot (q_0 + \alpha \cdot q_j) = Y_{i,j} \cdot q_0 + (Y_{i,j+1} + Y_{1,j+1} \cdot \alpha) \cdot q_j$$

which shows that the regression programm cannot find $Y_{i,j+1}$ and $Y_{i,j}$ separately, only $(Y_{i,j}+\alpha\cdot Y_{i,j+1})$ can be found. The same goes for H_0 $Y_{i,j}\cdot q_0$ To minimize this disturbing effect upon Ho, q(Gll) is added to q(921) rather than let it entirely out of the calculations. This leaves J = 9. In the calculation it was seen that two wells, H27 and Gl7, had no significant effect upon the waterlevel variation in any of the observation wells. This leaves J = 7.

The program operates in such a way, that it takes one variable in each step, into the regression. It calculates the standard error of the estimate and the T-value of each variable in each step. As a final result was selected the step where all the T's were less than \$\ddots\$ 1.0, and the standard error as close to minimum as possible. The result is presented in Table 3 and on Fig.11.

The input used were the monthly q-values as independent variables, and corresponding water levels, Hi in meters above sea level. The H's have to be interpreted as mean values over a month. But the water level curves are not continuous records, but 20-30 independent observations in each year. A few continuous records exist, and these show that significant fluctuations in water level can occur within a month (Fig. 5). The Hi's used here can therefore deviate by an unknown figure from the actual monthly mean values. The error caused by this is randomly distributed in time, and has therefore no effect on the regression.

In table 3, it is seen that the standard error of the estimate σ , is 4.4 - 7.3% of the range, or 6.36% on the average. The standard deviation of the H's was calculated to be very close to 25% of the range. This means that the regression has reduced the standard deviation to ca. 25% of its original value.

This indicates that it is reasonable to conclude that the drawdown is proportional to a time-integral function of the pumping rate. But this also shows a significant error of the estimate.

Looking at figure 10 it may be seen, that the deviation is mostly due to a time delay of order of magnitude 1 month. This is obviously due to the introduction of (9) instead of (7), and shows that the time delay αx is not negligible. This factor αx is called the time delay, because in actual calculation the integral in (7) would be calculated by summing up the monthly pumping rates to form a new function q(x,t)

$$q(x,t) = a_0 \cdot Q(t) + a_1 \cdot Q(t-\tau)$$

$$+ a_2 \cdot Q(t-2 \cdot \tau) + a_3 \cdot Q(t-3 \cdot \tau) + \cdot \cdot + a \cdot Q(t-n \cdot \tau)$$
 (13)

When summing monthly pumping rates

$$\tau = 1 \text{ month} = \frac{1}{12} \cdot \text{ year} = 2.628 \cdot 10^6 \text{ seconds.}$$

The a's are coefficients, dependent of x, t and the reservoir coefficients, but we always have maximum a, αx before t. That is

$$Max(a_n) = > n \cdot \tau = \alpha x$$

The q(t)'s in (10) are similar sums but there a_0 is always biggest.

On fig. 10 the coefficients Y_i, given in table 3 are plotted on semilog-paper, the abscissa being $Y_{i,j}$, the ordinate $R_{i,j}$ which is the distance from pumping well J to observation hole i in meters.

The points show a great scatter. But the coefficients for individual holes scatter much less. On fig. 10 the ranges of the wells with significant Y's in all 7 places are shown. Well no. 3 stands out as having much greater Y's than any other well, except one of two significant Y's for well no. 1 falls just within 3's range.

l and 3 are the deepest wells. No. 1 is about 1200 m deeper than the average depth of the other wells. That is a plausible explanation of its insignificance.

TABLE 3.

Name:	G 12	G 06	G 14	G 08	G 02	G 03	G 01	L
Hole no.	1	2	3	4	5	6	7	
H max.	20	19	15	20	19	24	22	Observed m
H min.	- 39	- 29	- 45	- 38	- 35	- 34	- 48	Observed m
ΔΗ	59	48	60	58	54	58	70	Range m
σ	4.09	3.29	2.63	3.45	3.52	4.05	5.11	St.err.m
σ%	6.9	6.9	4.4	5.9	6.5	7.0	7.3	100σ/ ΔΗ
Но	40.5	34.8	47.9	48.2	37.9	47.3	46.1	m
1	-	-	0.056	0.302	-	-	-	G 04
2	0.130	0.158	0.186	0.108	0.172	0.168	0.111	G 05 អ្ន
•3 0N 4	0.302	0.470	0.594	0.404	0.497	0.556	0.405	G 10
	-	-	0.231	0.124	-	0.102	-	G 15 0
Mell 6	0.158	-	-	0.118	0.055	-		G 198
× 6	0.063	0.047	0.156	0.102	0.035	0.072	0.105	G 20 m/1
7	0.141	0.101	0.105	0.114	0.109	0.121	0.161	921 H H

⁻ Means that coefficient is insignificant.

The explanation of the high coefficients of well no. 3 are the statistics of the data. The 7 observation series are very correlated. The lowest correlation coefficient is 0.950, the highest 0.996. The correlation coefficients between the 7 Q-series are in the range 0-0.5. This means, that a Yij's must be somewhat alike for each individual well, when found by multiple regression.

We can therefore summarize the following:

- 1. The reservoir contains a "storage volume", so the pressure drop in the aquifer is not proportional to any linear function of the pumping rates. A representation of the form (9), may be used for a definate interval of time.
- 2. A representation of the drawdown, according to (10) and (12) using the multiple regression technique, is sufficiently accurate for operational purposes (determining changes in the pressure cone that follow from minor changes in pumping rates).
- 3. A reliable estimate of the reservoir behaviour can only be obtained by solving the differential equation (4).

 (7) is an approximation to the complete solution and can be used for pumping from narrow wells.

5.2. Elastic Model.

5.2.1. Effects of pumping.

The response of the piezometric surface of the hydrothermal system to pumping is illustrated in figures 3, 4 and 5.

Figure 3 gives hydrographs of wells G7 and G1 for the jan. period /1966 to sept. 1969 along with monthly withdrawal of water from january 1965 to october 1969. It is constructed partly from monthly measurements and partly from automatic recordings of the water levels.

Figure 4 is a semilogaritmic plot of drawdown in well G12 caused by about 150 liters per second increase in pumping

rate during the period october 1967 to march 1968. It is constructed from an automatic record and emphasizes the straight line character of the drawdown plotted arithmetically against the logarithm of the time since the pumping rate was increased.

Figure 5 is an automatic record of the water level in well G 12 for february 1-9, 1968. It illustrates the response of the water level to intermittent pumping of wells G 15 and G 19, 11 and 24 liters per second, respectively, and to atmospheric pressure changes and oceanic tides.

The conclusions, summarized at the end of 5.1., along with the manner of response of water levels to pumping of long and short duration of time, atmospheric pressure changes and tides, illustrated in figures 3 to 5, suggest the volume elasticity of the aquifers and their artesian nature. Consequently Theis' expression from 1935 is applicable for determining aquifer characteristics and for predicting fluctuations of water levels due to past and future pumping.

Theis derived his expression for "the relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage" from analogy between laminar flow of ground-water and flow of heat through solid media. Later Jacob derived the same expression from pure hydrologic principles.

The Theis non equilibrium formula

$$h = h_0 - \frac{Q}{4 \pi}$$
 $\int_{11}^{\infty} \frac{e^{-u}}{u} du$, where $u = \frac{r^2 S}{4 \pi t}$

is actually a solution to the differential equation for unsteady two dimensional flow in an extensive aquifer:

$$\frac{d^2h}{dr^2} + \frac{1}{r} \frac{dh}{dr} = \frac{S}{T} \frac{dh}{dt}$$

for conditions of h = h_0 , at t = 0 and of h \rightarrow h_0 as r \rightarrow ∞ where ho is the head in the aquifer prior to pumping, Q is discharge of pumped well and h is head at distance r from pumping well at time t after beginning or end of pumping. T, the coefficient of transmissivity, is the rate of flow of

water at the prevailing temperature through a vertical strip of unit width, extending the height of the aquifer, under a hydraulic gradient of 100%. The coefficient of storage, S, is the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change of the component of head normal to that surface.

Jacob's expression for the coefficient of storage is

$$S = \theta y b \beta (1 + \frac{\alpha}{\theta \beta})$$

where b = thickness of aquifer

 α - compressibility of aquifer skeleton

 β - compressibility of water

y = unit weight of water

 θ = porosity of aquifer

The fraction of storage derived from the expansion of the water is $\theta yb\beta$ /S and that derived from compression of the aquifer $yb\alpha$ /S.

The non equilibrium formula assumes an infinite areal extent of a homogeneous and isotropic aquifer of constant thickness and withdrawal of water derived entirely from storage within the aquifer by compression of the aquifer skeleton by weight of overlying formations and expansion of the water with the lowering of pressure.

The exponential integral of the non-equilibrium formula can not be integrated directly but it can be expanded as the convergent series

$$W(u) = (-0.5772 - \ln u + u - \frac{u^2}{2.2} + \frac{u^3}{3.3} - \frac{u^4}{4.4} ...)$$

and simple approximate solutions obtained by graphical methods devised by Theis, Jacob and Chow.

The method of images, widely used in the theory of heat conduction in solids, enables the non equilibrium equation to be applied to aquifers of finite areal extent, like the Laugarnes aquifers, which appear to be impermeably bounded on two sides. Imaginary discharge wells are located across the impermeable boundaries to create a hydraulic system which is

equivalent to the effects of the boundaries, transforming the areally finite aquifers to ones of infinite areal extent. Methods of locating the image wells by pumping tests have been described by Ferris.

5.2.2. Aquifer characteristics and boundaries.

A number of tests have been made in wells in the Laugarnes area in order to determine values of the aquifer constants, T and S, and to locate impervious boundaries believed to exist between the three hydrothermal systems. The tests were conducted by observations of water levels in observation wells after a supply well was turned off or on, corrections being made for previous trends in water levels. Because of variation in demand the tests are of short duration, usually less than 10-20 hours and are interferred with by operating supply wells, the discharge of which varies somewhat by variations in water level.

Analyzed by the Theis graphical method of superposition the test data gave values of T ranging from 3.5 x 10^{-3} m²/sec, between wells G13 and G21, to 8.8 x 10^{-3} m²/sec, between wells G8 and G5 and values of S ranging from 3.9 x 10^{-5} between wells G7 and G20, to 3.2 x 10^{-4} between wells G12 and G19. Values of T and S between well G7 and six pumped wells are given below.

Pumped well	T m ² /sec	S
G 5	5.2×10^{-3}	1.9 x 10 ⁻⁴
G 9	5.9 x 10 ⁻³	1.2×10^{-4}
Gll	4.7×10^{-3}	4.0×10^{-5}
G19	5.2 x 10 ⁻³	2.3×10^{-4}
G20	5.8×10^{-3}	8.6×10^{-5}
G21	5.3 x 10 ⁻³	1.9×10^{-4}

By their manner of deviation from the Theis type curve, the test data indicate the presence of discharging image wells at a distance of about 2000 meters southwest of the well field or an impermeable vertical boundary striking northwest at half that distance. This location of the boundary is in agreement with well logs from wells G2, G3 and G6 and with the

anomalously steep slope of the piezometric surface towards the well field from the southwest (see fig.8). Tectonically, however, it disagrees with the surface geology of the Reykja-vik region to the east, which is characterized by numerous northeast striking step faults.

The short duration of the tests precluded attempts to locate the impermeable boundary believed to exist between the Laugarnes and Ellidaar hydrothermal systems. Its position, striking north to northeast about 2000 meters east of the Laugarnes well field, is however reasonably well defined by the number of available observation wells and by geological and geophysical data. Assuming straight line demarcations of the aquifers by the boundaries, their angle of intersection is of the order of 60 to 90 degrees.

An angle of intersection of 90 degrees between the boundaries demands three discharging image wells to satisfy the requirement of no flow across the boundaries, thereby simplifying the flow system to an aquifer of infinite areal extent with four discharging wells of equal strength. Similarly an angle of 60 degrees between the boundaries demands five discharging image wells.

5.2.3. Tidal effects.

Water levels in all but shallow wells in the Laugarnes area exhibit semidiurnal fluctuations, amplitudes (range ratios) diminishing inland from 11.0% of tidal amplitudes as recorded in Reykjavik harbour, in well G14, to 2.9% in well G11, 900 meters away from shore. The time lag increases from a few minutes in well G14 to 2 1/2 hours in well G11.

Ferris has shown that distances from a subaquaous outcrop of a nonleaky artesian aquifer, directly connected to a tidal body, plot as a straight line against the logarithm of the corresponding range ratios, the range ratio approaching unity as the distance approaches zero. From the slope of the plot he derived the relation $T/S = 0.59 \cdot x^2$

where

- T coefficient of transmissivity
- S coefficient of storage
- X = slope of plot
- to= period of the tide

Fig 6, which is a semilogartihmic plot of range ratios for six wells against their distances from shore, shows the range ratio as unity at a distance of -1280 meters or 1280 meters offshore. This is in disagreement with the time lag of a few minutes exhibited by well G14 and an indication of artesian aquifers effectively separated from the sea by an extensive confining layer, the tidal response being caused by changing load on the aquifers with the changing tide transmitted through the confining layer. Consequently the tidal efficiency, T.E., of the aquifers or the ratio of the range of water level fluctuation to the range of the tide, is about 11.8%.

The ratio T/S = 24.9 m²/sec, computed from the plot is of the same order of magnitude as that derived from pumping tests. The effects of earth tides, which elsewhere have been demonstrated by variations of water level of a few centimeters in artesian wells too far from the ocean to have any relation to it, have not been observed in the Laugarnes wells. By their phase relations to the effects of the oceanic tides they would, if present, tend to steepen the plot in figure 7, making the ratio T/S high. Leakance of the aquifers, to or from aquicludes, would on the other hand tend to flatten the plot making the computed value of T/S low.

5.2.4. Atmospheric pressure effects.

The mechanism in an artesian aquifer that produces response to changing tides also produces response to changes in atmospheric pressures. An increase in atmospheric pressure causes a decline of the water level while a decrease causes the water level to rise. The ratio of the change in water level to the corresponding change in atmospheric pressure is the barometric efficiency, B.E., of the aquifer.

Jacob has derived the following expressions relating the tidal and barometric efficiencies and the elasticity of an artesian aquifer:

T.E. =
$$\frac{\alpha/\theta\beta}{1 + \alpha/\theta\beta}$$
 and B.E. = $\frac{1}{1 + \alpha/\theta\beta}$

By relation to the storage coefficient

T.E. =
$$y b\alpha/S$$
 and B.E. = $\theta yb\beta/S$,

which are the fractions of storage derived from compression of the aquifer and expansion of the water, respectively.

The barometric efficiency observed in wells in the Laugarnes system ranges from 65% for shallow wells to about 85% for wells penetrating aquifer B. Assuming perfect elasticity for the Laugarnes aquifers and a tidal efficiency of 15% the theoretical percentage of storage attributable to the expansion of the water is 85% and that owing to compression of the aquifers 15%.

Figure 5 illustrates the response of the water level in well G12 to changes in atmospheric pressure. The barometric efficiency computed from the hydrograph is 82%.

5.2.5. Earthquakes.

Earthquake induced fluctuations of water levels have been observed in some wells equipped with recording gages in the Laugarnes system. Fig. 7 is a hydrograph of well G2 for may 15 and 16, 1968, showing fluctuations of water level caused by earthquake waves originating off the east coast of the island of Honshu, Japan, 74.2 degrees or 8250 km distant from Reykjavik. According to Stefansson, seismologist at the Icelandic Meteorological Service, the maximum fluctuations having a double amplitude of 20 centimeters reflect Rayleigh waves from the quake, which was of 8.2 Richter magnitude and originated at a depth of 10 kilometers at 00h 48m 55.4s G.M.T. on may 16. The expected single amplitude of the largest Rayleigh waves is of the order of 1 mm and their expected period is in excess of 40 seconds.

On this same occasion a maximum double amplitude of 40 cm was recorded in well H27 and 20 cm in well G7, while fluctuations of water level were not recorded in well G12.

5.2.6. Subsidence.

By combining Hooke's Law with Jacob's expression for the storage coefficient Lohman derived the following expression for the reduction in thickness of an elastic aquifer caused by reduction of hydrostatic pressure:

$$\Delta_b = \Delta_p \quad (\frac{S}{y} - \theta b \beta)$$

where Δp is the variation in artesian pressure and other symbols are as defined earlier.

Assuming a barometric efficiency of 85 per cant and a storage coefficient of 1.0 x 10^{-4} for the Laugarnes aquifers, the reduction in thickness resulting from a decline of the water level of 100 meters is computed as 0.16 centimeters. Even if withdrawal of water were solely attributable to compression of the aquifers, the reduction in thickness caused by elastic deformation would not exceed 1.0 centimeter.

Releveling made in 1961 and in october 1969 of benchmarks established in 1948-50 in the Laugarnes area revealed no relative movement of the bench marks in excess of the limits of accuracy of the levels which were computed as 3.9 mm/km for the 1948-50 survey and 1.4 mm/km for the 1961 survey, indicating the absence of any appreciable amount of plastic deformation of clay and hydrothermally altered tuff members of the aquifers and aquicludes.

5.2.7. Ground water inventory.

A quantitative evaluation of boundary effects and aquifer constants was made by comparing computed variation of water level in well G7 for the period march 10 to august 5, 1969, to observed water level at the end of the period. An angle of intersection of 90 degrees was assumed for the boundaries and a single discharging well was substituted for the well field for simplification.

TABLE 4.

Computed decline of water level Jan. 1957 to aug. 30, 1969.

Period	Days	aug.30, of withdrawal, on aug.30,69			water level	
		1969	Q, liters sec.	per ΔQ	- ∆ h	Δh
		1				
Jan. 57-sep. 59		4587	30	30	- 15.2	
oct.59-sep.62	1095	3587	60	30	- 15.2	
oct.62-mar.63	180	2492	100	40	- 19. 0	
apr.63-oct.63	195	2312	60	- 40		18.8
oct.63-mar.64	166	2117	150	90	- 41.8	
apr.64-sep.64	180	1951	70	- 80		36.8
oct.64-apr.65	210	1771	160	90	41.0	
may 65-oct.65	165	1561	75	- 85		38.0
oct.65-apr.66	180	1 396	203	128	57.0	
apr.66-oct.66	180	1216	75	-1 28		56.0
oct.66-may 67	203	1036	210	135	58.0	
may 67-oct.67	157	833	75	- 135		56.5
oct.67-may 68	210	676	225	15 0	61.2	
may68-sept.68	136	466	100	-1 25		48.0
oct.68-apr.69	210	330	270	170	62.8	
may 69-aug.30,	69-120	120	110	-1 60		50.3
					-371.2	304.4

Net computed decline of water level jan. 1, 1957 - aug. 30, 1969 - $\Sigma\Delta$ h = 66.8 meters.

The rate of pumping during the period was approximated by a series of 31 steps and the water level at the end of the period computed for various values of T and S and the four distances r_1 , r_2 , r_3 and r_4 from the observation well to the real discharging well and the image wells respectively. The computations, which were made by the aid of an IBM 1620 electronic computer, showed good agreement between computed and observed water levels for the values $T = 6.0 \times 10^{-3} \text{ m}^2/\text{sec}$, $S = 1.0 \times 10^{-4}$, $r_1 = 400 \text{ m}$, $r_2 = 2000 \text{ m}$, $r_3 = 4000 \text{ m}$ and $r_4 = 4400 \text{ m}$ (see figure 9).

The above values, which agree fairly well with those from short term pumping tests, were used to compute the fluctuations of water level in an observation well caused by past and future withdrawals of water from january 1957 through august 1974. Past withdrawals were averaged seasonally as given in table 4 and future withdrawals were arbitrarily assumed 280 liters per second for seven months and 100 liters per second for five months each year. The computed net decline of the water level from january 1957 until august 1969 and august 1974 is 66.8 meters and 73.0 meters respectively.

Although the elevation of the piezometric surface in the hydrothermal system in january 1957 is unknown, it may be approximately computed from observations of pressure made at a later date and withdrawals of water up to that date. From the observed shut in pressure of 2.5 kg/cm² of well Gl5 in october 1962 and rates of withdrawal as estimated in table 4, the decline of the water level from january 1957 until october 1962 is computed as 27 meters placing the piezometric surface in january 1957 at an elevation of 77 meters above sea level. Consequently the computed elevation of the piezometric surface in august 1969 is 10.2 meters above sea level and that for august 1974 4.0 meters above sea level, a decline of 6.2 meters between august 1969 and august 1974.

Observed water levels on august 5, 1969 were 9.71 meters above sea level in well G7 and 6.93 meters above sea level in well G12.

CONCLUSIONS.

The yearly withdrawal of 6-7 gigaliters of hot water from wells in the Laugarnes area is derived from three locally separate aquifers. The response of the aquifers' pressure heads to pumping, to changes in atmospheric pressure, tidal loading and distant earthquakes is in a manner approximating the principles of the Theis elastic theory, granting partial penetration of wells and boundary conditions.

Mathematical investigation of the water level fluctuation series has established that a linear relationship between a time-integral of the pumping rate and the drawdown curves can be used, provided that a standard error of about 2 m is allowed. This kind of model is usable for several years without charging the coefficients, but this time can be lengthened and the overall accuracy improved by using more complicated relationships than simple linearity. The initial response of the aquifers, to sudden changes of pumping rates, follow the Theis nonequilibrium formula very well, and both methods conclude, that no significant increase in overall drawdown should take place in the coming years, provided that the pumping is not increased.

The relatively small amount, less than 15%, of water theoretically derived from compression of the aquifers emphazises the rugged character of the basalt formations. Location of impermeable boundaries indicated by long and short term hydrographs has not been definitely established, but a model of essentially horizontal artesian aquifers with two impermeable boundaries at approximately right angles is compatible to hydrological and geological evidence.

Evidence of recharge to the aquifers from intake areas or from leakance through aquicludes is not detectable in hydrographs, but may very well be masked by uneven and cyclic pumping. The absence of recharge to the aquifers would indicate an areal extent counted in tens of thousands of km². Controlled pumping tests of long duration are likely to detect leakance, if any, and assist in location of boundaries, and recharge areas.

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EXPLANATION TO FIGURES.

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- Fig. 3. Hydrographs of wells G7 and G16 and monthly with-drawals of water 1965 to 1969.
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